VU University Amsterdam	Calculus 1, Second Test
Faculty of Sciences	19-10-2015
Department of Mathematics	12.00 - 2.00 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = x\sqrt{x^2 + 3}.$$

   a) Prove that f is one-to-one on $\mathbb{R}$.
   b) Prove that f has an inverse function f^{-1} with domain $\mathbb{R}$.
   c) Calculate $(f^{-1})'(0)$.

2. Consider the function $f : (0, e] \to \mathbb{R}$ defined by

$$f(x) = x^2 \ln(x).$$

   a) Find the absolute extreme values of f on $(0, e]$.
   b) Calculate the inflection point(s) of the curve $y = f(x)$ on $(0, e)$.

3. Calculate
   a) $\lim_{x\to\infty} \Big(x^2 - \ln(x^2 e^x)\Big)$.
   b) $\lim_{x\to 0} \Big(1 + \sin(x)\Big)^{1/x}$.

4. a) Find $P_2(x)$, the second-order Taylor polynomial of $f(x) = \frac{1}{x}$ about $x = 1$.
   b) Approximate $\frac{1}{1.02}$ with the formula derived in part a) and show that the absolute value of the error is less than $(0.02)^3 = 0.000008$.

(Please turn over)

5. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = 3x \int_4^{x^2} e^{-\sqrt{t}}\, dt.$$

Calculate $f'(2)$.

6. Calculate

a) $\displaystyle\int \sqrt{x} \cos(\sqrt{x})\, dx.$

b) $\displaystyle\int \frac{x}{x^2 + x - 20}\, dx.$

c) $\displaystyle\int_5^{\infty} \frac{1}{x^2 + 25}\, dx.$

Scoring:

1 : a) $1\frac{1}{2}$	2 : a) 3	3 : a) 2	4 : a) 2	5 : 2	6 : a) 3
b) $1\frac{1}{2}$	b) 2	b) 2	b) 2		b) 2
c) 2					c) 2
5	5	4	4	2	7

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$