VU University Amsterdam	Calculus 1, First Test
Faculty of Sciences	22-09-2015
Department of Mathematics	9.00 am - 11.00 am

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine all x which satisfy the inequality

$$\frac{|x+3|}{2x+1} < 2.$$

2. The function $f : D_f \to \mathbb{R}$ is defined as

$$f(x) = \frac{1}{1-\sqrt{x-2}}.$$

Find the (maximal) domain D_f and the range R_f of f.
[Explain your answers!]

3. Calculate the following limits:

a) $\lim\limits_{x\to 1} \dfrac{x^2-1}{\sqrt{x+3}-2}$.

b) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{9x^2-5x+3}}{3x-7}$.

c) $\lim\limits_{x\to 0} \dfrac{2x-\sin(3x)}{\tan(x)+4x}$.

4. The function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \begin{cases} \cos\left(x-\frac{\pi}{4}\right) & \text{if } x \geq \pi, \\ x+k & \text{if } x < \pi. \end{cases}$$

For which value of k is f continuous at $x = \pi$? [Explain!]

(Please turn over)

5. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ \\ 0 & \text{if } x = 0. \end{cases}$$

Use the definition of derivative to prove that f is differentiable at $x = 0$ and calculate $f'(0)$. [Note: explain all your steps!]

6. A curve is implicitly given by the equation

$$xy + y^3 = 2x^2.$$

Find the equation of the tangent line to the curve at $(x, y) = (1, 1)$.

7. Prove, using the Mean Value Theorem, that for all $x \in (0, \frac{1}{4}\pi)$:

$$\tan(x) < 2x.$$

Scoring:

1 : 3	2 : 4	3 : a) 3 b) 2 c) 3	4 : 2	5 : 3	6 : 3	7 : 4
3	4	8	2	3	3	4

$$\text{Final grade} = \frac{\#\text{ points}}{3} + 1$$