VU University Amsterdam	Calculus 1, Resit
Faculty of Sciences	06-01-2015
Department of Mathematics	18.30 - 21.15 pm

The use of a calculator, a book, or lecture notes is not permitted.
Do not just give answers, but give calculations and explain your steps.

1. Determine all x which satisfy the inequality

$$\frac{2x-5}{2-x} \leq 2.$$

2. Calculate the following limits:

   a) $\lim_{x\to 0}(3x^2-3x+1)^{\frac{1}{x}}$.

   b) $\lim_{x\to 0}\dfrac{\sqrt{9-x}-\sqrt{9-3x}}{x}$.

3. Consider the curve

$$\sin(3xy) = y.$$

   a) Determine $\frac{dy}{dx}$ in terms of x and y.

   b) Find the equation of the tangent to this curve at $(x,y) = (\frac{1}{9}\pi, \frac{1}{2})$.

4. The function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \begin{cases} \dfrac{\ln(1+3x)}{2x} & \text{if } x > 0; \\ \\ ax+b & \text{if } x \leq 0. \end{cases}$$

   a) For which values of a and b is $f(x)$ continuous in 0?

   b) For which values of a and b is $f(x)$ differentiable in 0?

(Please turn over)

5. Find $P_3(x)$, the third-order Taylor polynomial of the function $f(x) = e^{2x}$ about $x = -2$.

6. Use the Mean Value Theorem to prove that for all $x > 0$

$$\sqrt{2+3x} < \sqrt{2} + \frac{3x}{2\sqrt{2}}.$$

7. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = 2x \int_{\frac{\pi}{4}}^{\arctan(x)} \tan^2(t)\, dt.$$

Calculate $f'(1)$.

8. Calculate

a) $\displaystyle\int_0^1 \frac{x}{\sqrt{3x^2+1}}\, dx.$

b) $\displaystyle\int x^2 e^x\, dx.$

c) $\displaystyle\int \frac{x^2}{x^2+5x+6}\, dx.$

9. Determine if the following improper integral is convergent or divergent. Motivate your answer.

$$\int_1^\infty \frac{1+\ln x}{x}\, dx.$$

Scoring:

1 : 3	2 : a) 3 b) 3	3 : a) 3 b) 3	4 : a) 3 b) 4	5 : 3	6 : 4	7 : 3	8 : a) 3 b) 3 c) 4	9 : 3
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3	6	6	7	3	4	3	10	3

$$\text{Final grade} = \frac{\#\text{ points}}{5} + 1$$