VU University Amsterdam	Calculus 1
Faculty of Sciences	First Test
Department of Mathematics	24-09-2014, 13:30-15:30 uur

The use of a calculator, the book, formula tables or lecture notes is <u>not</u> permitted

1. Solve:
   a) $\dfrac{3-2x}{x-1} < 1.$
   b) $|x+2| \geq |x-5|.$

2. Calculate the following limits:
   a) $\lim\limits_{x\to -3} \dfrac{3x^2+8x-3}{x+3}.$
   b) $\lim\limits_{x\to\infty} (\sqrt{9x^2-5x+3}-3x).$
   c) $\lim\limits_{x\to 1} (x-1)^2 \cos\left(\dfrac{1}{x-1}\right).$

3. Prove that the following equation has a real root:
$$x^3 - 3x^2 = 2.$$

4. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by:
$$f(x) = \begin{cases} \sin(3x - \frac{1}{3}\pi) + 1 & \text{als } x > 0 \\ ax + b & \text{als } x \leq 0. \end{cases}$$
   a) For which values of a and b is f continuous at 0? Explain your answer.
   b) For which values of a and b is f differentiable at 0? Explain your answer.

5. Consider the curve:

$$xy^2 + \tan(3x^2) = 4.$$

Calculate $\dfrac{dy}{dx}$ in terms of x and y.

6. Proof, using the Mean Value Theorem, that for all $x > 0$:

$$\sqrt[3]{1+2x} < 1 + \frac{2}{3}x.$$

Scoring:

1 : a) 3 b) 3	2 : a) 3 b) 3 c) 4	3 : 4	4 : a) 4 b) 4	5 : 4	6 : 4
6	10	4	8	4	4

$$\text{Final grade} = \frac{\#\text{ points}}{4} + 1$$