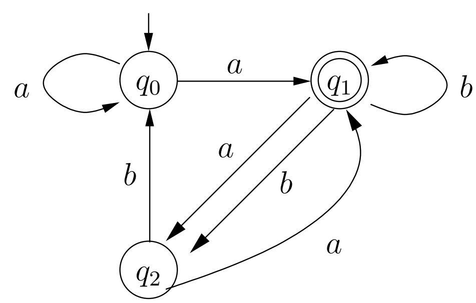
**Exam Automata & Complexity**

Vrije Universiteit, 25 March 2015, 12:00-14:45

*(This exam consists of 90 points in total; every student gets 10 points bonus.)*

*(At the exam, copies of slides can be used, without handwritten comments. The textbook by Linz, handouts, and laptop are* not *allowed!)*

1. Consider the nfa




   (a) Transform this nfa into a dfa, with as states subsets of $\{q_0, q_1, q_2\}$.
   (States of this dfa that are not reachable from $\{q_0\}$ can be omitted. But the trap state must be included.) (8 pts)

   (b) Perform the minimisation algorithm for dfa's on the resulting dfa.
   (Give explicitly all intermediate steps and splitting criteria of the reduction from the original dfa to the minimal dfa.) (10 pts)

2. Check using the string matching algorithm whether *baabbabab* contains a substring that is in $L(a^*(ba^* + ab^*)(ab)^*a)$.

   (Describe the entire construction: the corresponding nfa, and the on-the-fly construction of the corresponding dfa.) (12 pts)

3. Is the language $\{a^n b^{2n} a^n \mid n \geq 0\}$ context-free? If yes, give a context-free grammar that produces this language. If no, show this by means of the pumping lemma. (12 pts)

4. Show that the context-free grammar

$$S \rightarrow bSa \mid cSa \mid \lambda$$

is LL(1). (Also give the needed FIRST and FOLLOW collections.)

Determine using the parsing table whether $bca$ and $bcaa$ are in the corresponding language (12 pts)

5. Draw an npda $M$ such that $L(M)$ consists of all strings over $\{a, b\}$ with odd length and $a$ as symbol in the middle. (8 ptn)

6. Given is the grammar $G$ with as productions

$$\begin{array}{lll} S & \rightarrow & AB \\ A & \rightarrow & AB \mid BA \\ B & \rightarrow & AA \mid BB \end{array} \qquad \begin{array}{lll} & & \\ AA & \rightarrow & a \\ AB & \rightarrow & b \end{array}$$

(a) Transform the question whether string $ab$ is in $L(G)$ into an instance of the Modified Post Correspondence Problem. (4 pts)

(b) Give a derivation of $ab$ using the productions of $G$.

Transform this derivation into a solution for the corresponding instance of the MPCP. (10 pts)

7. Let $f : \{0, 1\}^2 \rightarrow \{0, 1\}^2$ be defined as follows:

$$\begin{array}{lllll} f(00) & = & f(10) & = & 11 \\ f(01) & = & f(11) & = & 00 \end{array}$$

Apply Simon's algorithm to determine a linear dependence between the digits of $s = 10$. (Give one possible scenario.) (14 pts)