Exam Automata & complexiteit

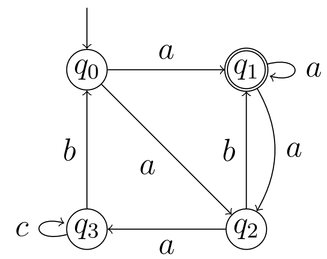
Vrije Universiteit, 26 March 2014, 12:00–14:45

Remarks. *Copies of the slides without handwritten notes may be used for this exam. Linz's textbook and laptop may* ***not*** *be used! This exam consists of 6 questions on 2 pages. In total 90 points can be scored; every student gets a bonus of 10 points. Good luck!*

1. (a) (5 points) Give the minimal dfa that describes the language of strings over $\{a, b\}$ in which the number of as in the string is not divisible by three.

   (b) (5 points) Convert this dfa into a right linear grammar that describes the same language.

2. Consider the nfa



   (a) (8 points) Convert this nfa into a dfa, with subsets of $\{q_0, q_1, q_2, q_3\}$ as states. (States of this dfa that are not reachable from $\{q_0\}$ can be omitted. However, it is not allowed to omit the trap state.)

   (b) (10 points) Apply the minimisation algorithm for dfas to the resulting dfa. (Explicitly give all intermediate steps and splitting criteria of the reduction of the original dfa to the resulting minimal dfa.)

3. $\Sigma = \{a, b, c\}$. Let $n_a(w)$, $n_b(w)$ and $n_c(w)$ denote the number of as, bs and cs in string w, respectively.

   (a) (12 points) Show that the language $L = \{w \mid n_a(w) + n_b(w) = n_c(w)\}$ is not regular. Use the pumping lemma.

   (b) (8 points) Show that the language $L = \{w \mid n_a(w) + n_b(w) = n_c(w)\}$ is context free.

4. (12 points) Show that the context free grammar

$$\begin{array}{rcl} S & \rightarrow & aAB \mid BAa \\ B & \rightarrow & b \\ A & \rightarrow & cS \mid \lambda \end{array}$$

   is LL(1). (Also give the required FIRST and FOLLOW sets.)

   Determine using the parse table whether $aacabb$ is in the corresponding language.

5. Consider the Turing machine M with $\Sigma = \{a, b\}$, $\Gamma = \Sigma \cup \{\Box\}$, $F = \{q_2\}$ en

$$\delta(q_0, a) = \{(q_0, b, R)\} \qquad \delta(q_0, a) = \{(q_1, b, R)\} \qquad \delta(q_1, b) = \{(q_2, b, L)\}$$

   (a) (8 points) Show how the question $x \in L(M)$ reduces to (an instance of) the bounded tiling problem, and apply this reduction to the following two strings: $x = aba$ and $x = baa$.

   (b) (7 points) Describe the language $L(M)$ and give an unrestricted grammar G such that $L(G) = L(M)$.

6. (15 points) Let $f : \{0, 1\}^2 \rightarrow \{0, 1\}^2$ be defined as follows:

$$\begin{array}{rcccl} f(00) & = & f(10) & = & 01 \\ f(01) & = & f(11) & = & 10 \end{array}$$

   Apply Simon's algorithm to determine a linear dependency for the digits of $s = 10$. (Give a possible scenario.)