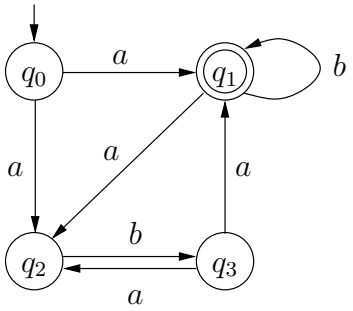
Exam Automata & Complexity

Vrije Universiteit, 27 March 2013, 12:00-14:45

(This exam consists of 90 points in total; every student gets 10 points bonus.)

(At the exam, copies of slides can be used, without handwritten comments. The textbook by Linz, handouts, and laptop are not *allowed!)*

1. Transform the nfa below into an equivalent dfa, with as states subsets of $\{q_0, q_1, q_2, q_3\}$.



   (States of this dfa that are not reachable from $\{q_0\}$ can be omitted. Give all reachable states, including $\emptyset$.) (10 pts)

2. (a) Show that $\{vwv \mid v, w \in \{a, b\}^*, |v| = 2\}$ is a regular language. (6 pts)

   (b) Show using the Pumping Lemma that $\{vwv \mid v, w \in \{a, b\}^*, |w| = 2\}$ is not a regular language. (10 pts)

3. Show that the following context-free grammar is ambiguous:

$$\begin{array}{lcl} \textit{Statement} & \rightarrow & \texttt{if } \textit{Condition} \texttt{ then } \textit{Statement} \quad | \\ & & \texttt{if } \textit{Condition} \texttt{ then } \textit{Statement} \texttt{ else } \textit{Statement} \quad | \\ & & x := 3 \quad | \quad x := 4 \\ \textit{Condition} & \rightarrow & x < 5 \quad | \quad x > 9 \end{array}$$

   Here *Condition* and *Statement* are variables, and `if`, `then`, `else`, $x := 3$, $x := 4$, $x < 5$ and $x > 9$ terminals. (8 pts)

4. Show that the context-free grammar

$$\begin{aligned} S &\rightarrow aAb \mid bAa \\ A &\rightarrow cS \mid \lambda \end{aligned}$$

is LL(1). (Also give the necessary FIRST and FOLLOW sets.)

Determine using the parsing table whether *aacabb* is in the corresponding language. (12 pts)

5. $\Sigma = \{a, b\}$, and $n_a(w)$ and $n_b(w)$ represent the number of a's and b's in string w, respectively. Give an npda that accepts the following language:

$$\{w \mid n_a(w) < n_b(w)\}$$

(12 pts)

6. Given is the grammar G with as productions

$$\begin{aligned} S &\rightarrow AB & \qquad AA &\rightarrow a \\ A &\rightarrow AB \mid BA & \qquad AB &\rightarrow b \\ B &\rightarrow AA \mid BB \end{aligned}$$

   (a) Transform the question whether string ab is in $L(G)$ into an instance of the Modified Post Correspondence Problem. (6 pts)

   (b) Give a derivation of ab using the productions of G.

   Transform this derivation into a solution for the corresponding instance of the MPCP. (10 pts)

7. Let $f : \{0,1\}^2 \rightarrow \{0,1\}^2$ be defined as follows:

$$\begin{aligned} f(00) &= f(11) = 10 \\ f(01) &= f(10) = 01 \end{aligned}$$

Apply Simon's algorithm to determine a linear dependence between the digits of $s = 11$. (Give one possible scenario.) (16 pts)