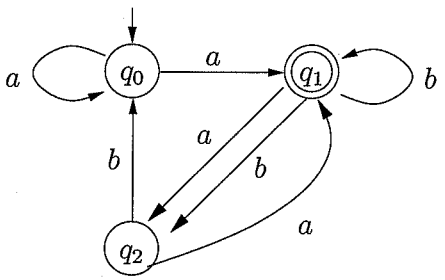
Exam Automata & Complexity

Vrije Universiteit, 27 October 2010, 8:45-11:30

(This exam consists of 90 points in total; every student gets 10 points bonus.)

(At the exam, copies of slides can be used, without handwritten comments. The textbook by Linz, handouts, and laptop are not *allowed!)*

1. Consider the nfa



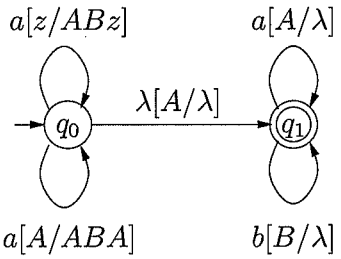
   (a) Transform this nfa into a dfa, with as states subsets of $\{q_0, q_1, q_2\}$.
   (States of this dfa that are not reachable from $\{q_0\}$ can be omitted.) (6 pts)

   (b) Perform the minimisation algorithm for dfa's on the resulting dfa.
   (Give explicitly all intermediate steps and splitting criteria of the reduction from the original dfa to the minimal dfa.) (12 pts)

2. Check using the string matching algorithm whether *baabbabab* contains a substring that is in $L(a^*(ba^* + ab^*)(ab)^*a)$.

   (Describe the entire construction: the corresponding nfa, and the on-the-fly construction of the corresponding dfa). (12 pts)

3. Consider the following npda M (with $\Sigma = \{a, b\}$ and $\Gamma = \{z, A, B\}$ and stack-startsymbol z):



(a) Which language is accepted by M? (4 pts)

(b) Construct a context-free grammar G with $L(G) = L(M)$.
(Make sure the end state is always reached with an empty stack, and use variables $(q\,c\,r)$; also give the start variabele.) (12 pts)

4. Show that the context-free grammar

$$S \rightarrow bSa \mid cSa \mid \lambda$$

is LL(1). (Also give the needed FIRST and FOLLOW collections.)
Determine using the parsing table whether *bca* and *bcaa* are in the corresponding language (12 pts)

5. Is the question whether a recursively enumerable language is empty decidable?
(Motivate your answer.) (7 pts)

6. Show that the class P is closed under union, intersection and complement. (10 pts)

7. Let $f : \{0,1\}^2 \rightarrow \{0,1\}^2$ be defined as follows:

$$\begin{array}{lllll} f(00) & = & f(01) & = & 00 \\ f(10) & = & f(11) & = & 01 \end{array}$$

Perform Simon's algorithm to determine a linear dependency of the digits of $s = 01$. (Give one possible scenario.) (15 pts)