vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Asset Pricing 4.1
Code:	E_FIN_AP
Coordinator:	Frode Brevik
Date:	December 12, 2013
Time:	08.45–11.30
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question is worth 0.5 points. A total of 10 points can be earned. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than Thursday, January 10, 2014.
Inspection:	Wednesday, January 15, 2014, 09:30–10:30. Room to be announced.
Number of pages:	7 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her wealth optimally among the stocks of two companies, stock 1, 2, and a risk-free bond. Expected returns are given by

$$R_t^f = 1.02 \qquad E_t[R_{t+1}] = \begin{bmatrix} R_{t+1}^1 \\ R_{t+1}^2 \end{bmatrix} = \begin{bmatrix} 1.08 \\ 1.14 \end{bmatrix}$$

The returns to the two stocks are uncorrelated and they both have a standard deviation of 0.3. The investor chooses a vector of portfolio weights ω for the two stocks to maximize:

$$E[R_{t+1}^p] - \frac{\gamma}{2}\sigma^2(R_{t+1}^p)$$

where $E[R_{t+1}^p]$ and $\sigma^2(R_{t+1}^p)$ are the expected return and the variance of the return to the investors portfolio.

(a) What percentage of her wealth should she allocate to each of the two risky assets if her risk aversion is $\gamma = 1$?

(b) Compute the expected return and standard deviation of the investor's portfolio.

(c) Compute the expected return and standard deviation of the return to the tangency portfolio between the Risky-Asset Frontier and the Mean-Variance Frontier.

(d) Sketch the Risky-asset frontier and the Mean-Variance Frontier in the standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

i. The investor's portfolio.

ii. The tangency portfolio.

iii. Risk-free bonds.

iv. The two stocks.

2. Use the same assets, expected returns and standard deviations as in the first exercise, but assume that there are two investors: The first investor can only take positions in the first asset, while the second can only take positions in the second asset. The first investor has a wealth of 600 and a risk-aversion parameter $\gamma = 1$, while the second investor has a wealth of 300 and risk-aversion parameter $\gamma = 2$. The market capitalization of the first asset is 400 and that of the second asset is 200.

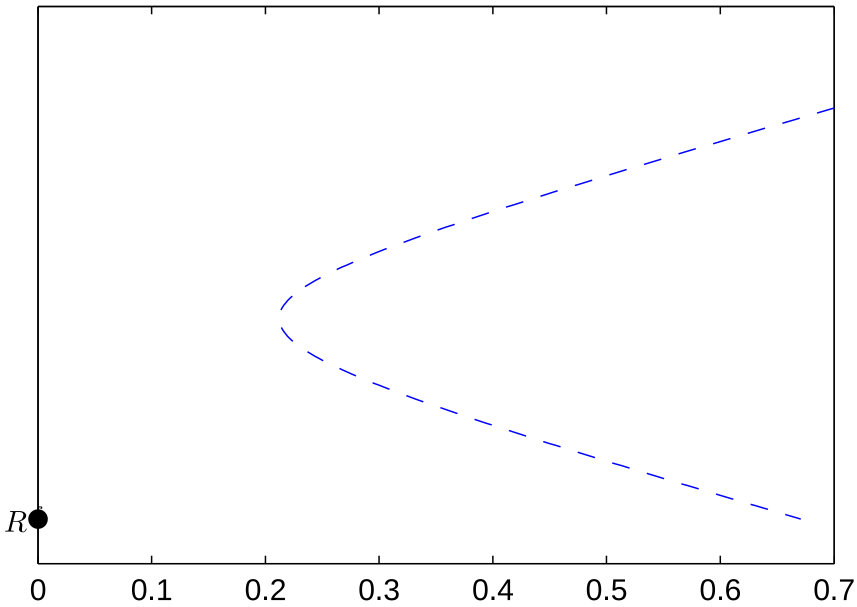
(a) Find the optimal portfolio of each of the investors and check whether the markets for each of the two stocks are in equilibrium. (Demand = Supply.)

(b) Find the betas of the two assets and check whether the CAPM holds.

(c) Now assume that markets are liberalized so that both investors are allowed to take positions in both stocks. Show that there is excess demand for both stocks at the current expected returns and variances.

(d) Show that a equilibrium obtains if the expected excess return of the first stock goes down to 4.8% and the expected excess return to the second stock goes down to 2.4 %, with no change in the variances.

3. The figure below shows a typical Risky-Asset Frontier together with the risk-free rate:



Copy the figure to your solution sheet, and, in the same figure, draw thee upper part of the Mean-Variance Frontier for an investor who is only allowed to invest up to 75% of her wealth in risky assets.

4. The following table excerpt (taken from Frazzini and Pedersen, 2013) shows the performance of portfolios of stocks sorted by their estimated betas. P1 is the return of the equal weighted portfolio of the stocks from the decile with the lowest betas. P10 is the return of the equal weighted portfolio of stocks in the decile with the highest betas.

	P1 (Low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)
Excess return	**0.91**	**0.98**	**1.00**	**1.03**	**1.05**	**1.10**	**1.05**	**1.08**	**1.06**	**0.97**
	(6.37)	(5.73)	(5.16)	(4.88)	(4.49)	(4.37)	(3.84)	(3.74)	(3.27)	(2.55)
CAPM alpha	**0.52**	**0.48**	**0.42**	**0.39**	**0.34**	**0.34**	**0.22**	0.21	0.10	-0.10
	(6.30)	(5.99)	(4.91)	(4.43)	(3.51)	(3.20)	(1.94)	(1.72)	(0.67)	-(0.48)
3-factor alpha	**0.40**	**0.35**	**0.26**	**0.21**	**0.13**	**0.11**	-0.03	-0.06	**-0.22**	**-0.49**
	(6.25)	(5.95)	(4.76)	(4.13)	(2.49)	(1.94)	-(0.59)	-(1.02)	-(2.81)	-(3.68)
4-factor alpha	**0.40**	**0.37**	**0.30**	**0.25**	**0.18**	**0.20**	**0.09**	**0.11**	0.01	-0.13
	(6.05)	(6.13)	(5.36)	(4.92)	(3.27)	(3.63)	(1.63)	(1.94)	(0.12)	-(1.01)
5-factor alpha*	**0.37**	**0.37**	**0.33**	**0.30**	**0.17**	**0.20**	**0.11**	0.14	0.02	0.00
	(4.54)	(4.66)	(4.50)	(4.40)	(2.44)	(2.71)	(1.40)	(1.65)	(0.21)	-(0.01)
Beta (ex ante)	0.64	0.79	0.88	0.97	1.05	1.12	1.21	1.31	1.44	1.70

(a) Find (i) the expected return, (ii) the CAPM alpha, and (iii) the beta of the long short strategy:

$$R^{long-short} = R^{P1} - R^{P10}$$

(For each dollar you go long in P1, you go one dollar short in P10.)

(b) The preferred strategy of Frazinni and Pedersen is the following:

$$R^{BAB} = \frac{1}{\beta_{P1}}\left(R^{P1} - R^{f}\right) - \frac{1}{\beta_{P10}}\left(R^{P10} - R^{f}\right)$$

Find (i) the expected return, (ii) the CAPM alpha, and (iii) the beta of this strategy.

5. Acharaya-Pedersen (2005) assume that every investor is a mean-variance optimizer with an investment horizon of 1 period and show that, in equilibrium, there is a CAPM relation for net returns:

$$R^{i,net}_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - X^i_{t+1}$$

where X^i_{t+1} is the stochastic transaction cost for trading stock i at $t+1$, measured as a percentage of the price at time t. The equilibrium requires:

$$E_t[R^i_{t+1} - X^i_{t+1}] = R^f + \frac{\mathrm{cov}(R^i_{t+1} - X_{t+1}, R^m_{t+1} - X^m_{t+1})}{\mathrm{var}(R^m_{t+1} - X^m_{t+1})}\lambda_t$$

where

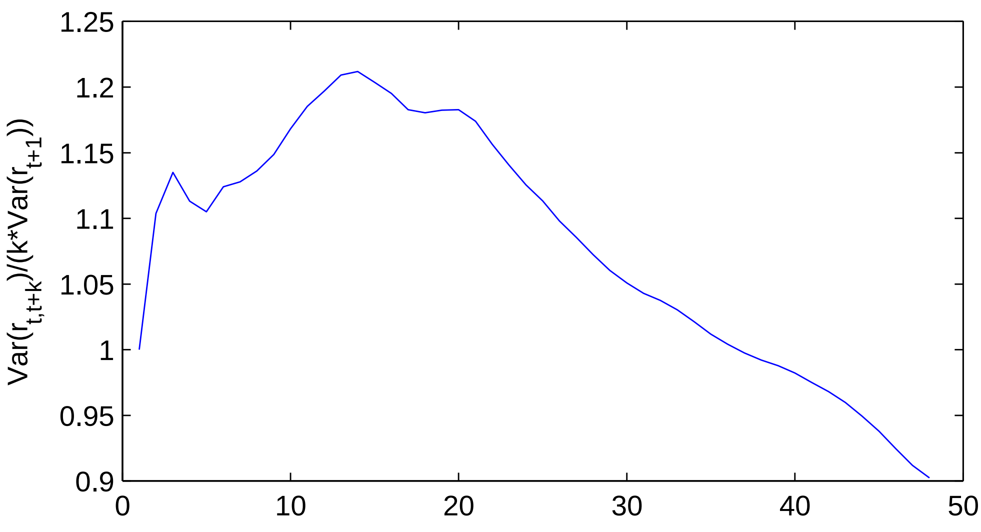
$$\lambda_t = E_t\left[R^m_{t+1} - X^m_{t+1} - R^f\right]$$

and R^m_{t+1} and X^m_{t+1} is the realized return to the market portfolio and the transaction cost for trading the market portfolio, respectively:

(a) Decompose this relation to show that the expected return depends on the covariance of the asset returns with the return to the market portfolio and 3 liquidity risks.

(b) Give a short economic interpretation for each of the 3 liquidity risks.

6. The figure below, taken from the lecture notes, shows the variance ratio statistic different holding periods for the value weighted returns of the US stock market from 1925-2012.



What does the figure tell you about mean-reversion and momentum for different holding periods for the aggregate stock market?

7. Campbell and Shiller decompose unexpected stock returns into

$$r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{t+1+j}$$

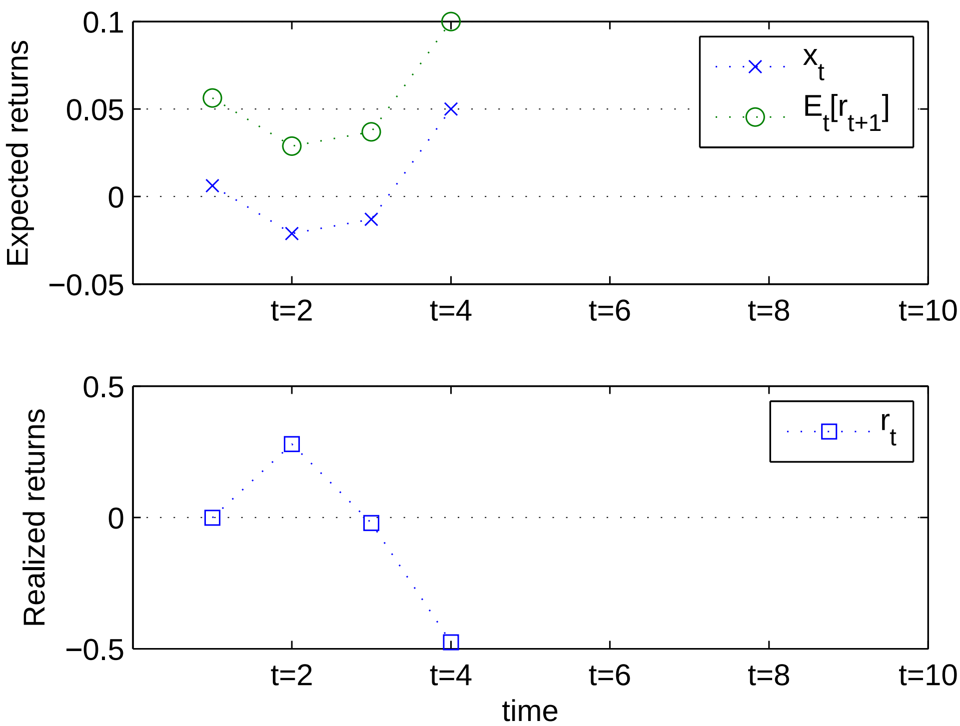
Let $\rho = 0.98$, the expected log dividend growth constant and equal to 2%, and required stock returns are mean reverting around a long term average value of 4.5%:

$$\Delta d_{t+1} = 0.02$$
$$E_t[r_{t+1}] = 0.045 + x_t,$$

where the time varying component of required stock returns x_t follows the mean zero AR(1) process

$$x_t = (0.9)x_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, (0.02)^2)$$

The following figure gives a sequence of realizations of the time-varying required return component x_t as well as corresponding required and realized returns.



(a) In the figure, there appears to be a negative relation between changes in required returns for $t+1$ (top panel) and realized returns for t (bottom panel). Explain economically why this makes sense.

(b) Copy the top panel of the figure to your answer sheet and add the expected future values of x_t to the figure. That is, add the series

$$E_t[x_{t+j}], \qquad \text{for } t = 4 \text{ and } j = 1, 2, \ldots, 6.$$

(c) Assume, that at time $t = 5$, we have $\epsilon_t = -0.05$:

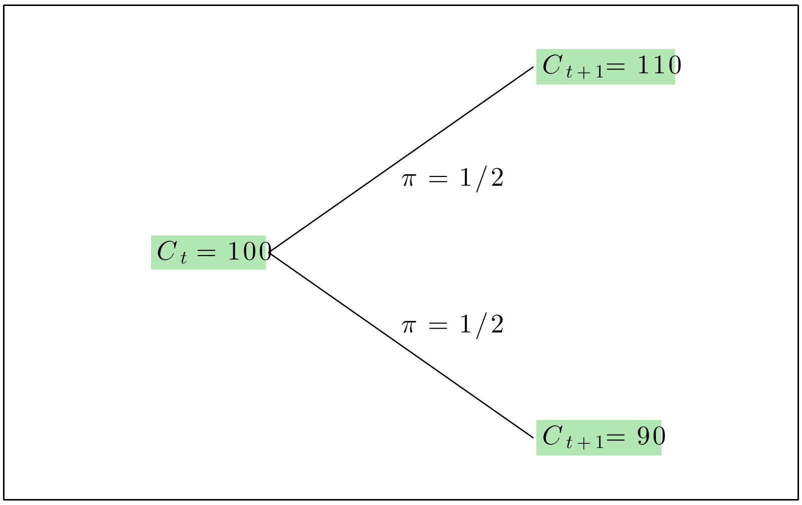
i. Find x_t for $t = 5$ using the process for x and the information in the figure.

ii. Find r_t for $t = 5$ using the Campbell-Shiller decomposition.

8. Consider the following simple economy, where C denotes consumption and π the probability of going to each state. The probability of going to the up state next period is equal to 1/2. Assume the representative (typical) investor has a utility function over consumption C_t given

$$U(C) = \ln C,$$

and a time discount factor of $\theta = 0.95$.



(a) Assume that the values for C_t and C_{t+1} given in the figure are the equilibrium consumption levels of the representative investor. Find the implied values of:

i. The marginal utility of the representative investor at t and in both states at time $t+1$.

ii. The stochastic discount factor between time t and $t+1$ for both possible states of the economy, m_{t+1}.

(b) Find (1) the current prices and (2) the expected returns of the following 3 securities.

i. A security that pays out 2 in the upstate at $t+1$, and nothing in the downstate at $t+1$.

ii. A security that pays out 2 in the downstate at $t+1$, and nothing in the upstate at $t+1$

iii. A security that pays out 1 in both the upstate and the downstate at time $t+1$.

(c) Explain the difference in prices of the three securities economically using the general pricing formula

$$p(x_{t+1}) = \frac{E_t[x_{t+1}]}{R_t^f} + \text{cov}_t(m_{t+1}, x_{t+1}),$$

where x denotes the risky payoff, m_{t+1} the stochastic discount factor, and R_t^f the one period risk-free rate.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \qquad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - (E[x])^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

$$x_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

$$E_t[x_{t+k}] = \mu + \theta_k \epsilon_t + \cdots \theta_q \epsilon_{t+k-q} \qquad \text{(conditional expectation)}$$
$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process with zero mean

$$x_t = \phi x_{t-1} + \epsilon_t \qquad \text{with } |\phi| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

$$E[x_t] = 0 \qquad \text{(unconditional expectation)}$$
$$E_t[x_{t+j}] = \phi^j x_t \qquad \text{(conditional expectation)}$$
$$\text{var}\,(x_t) = \frac{1}{1 - \phi^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\phi^j}{1 - \phi^2}\sigma^2 \qquad \text{(auto-covariance)}$$