Solution Key, December 2013

1. (a)

$$\omega = \frac{1}{\gamma}\Omega^{-1}E[R^e] = \frac{1}{1}\begin{bmatrix}11.1111 & 0\\ 0 & 11.1111\end{bmatrix}\begin{bmatrix}0.06\\0.12\end{bmatrix} = \begin{bmatrix}0.666667\\1.33333\end{bmatrix}$$

So 67 % to the first asset and 133 % to the second.

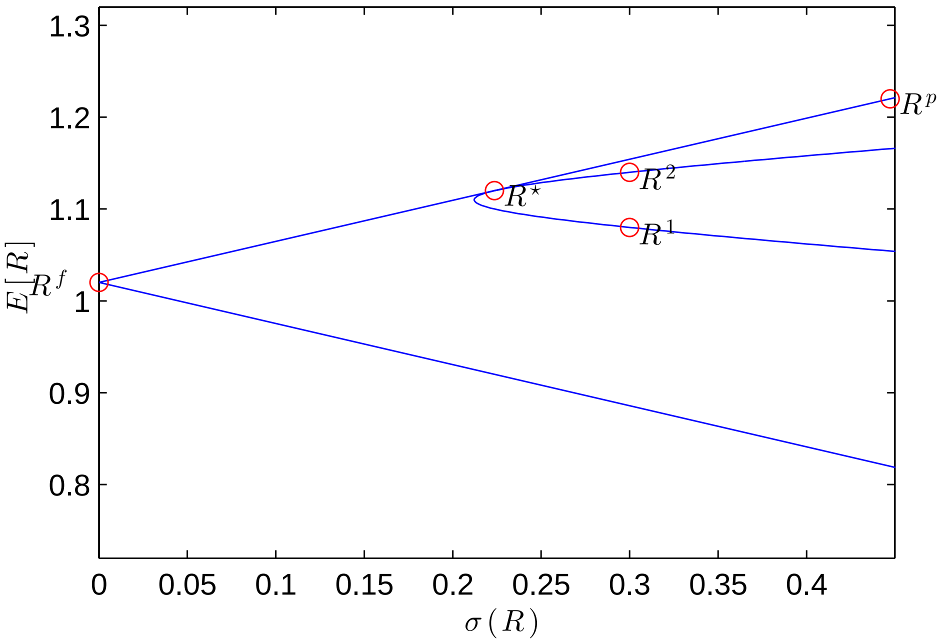
(b)

$$E[R^p] = R^f + \omega' E[R^e] = 1.22$$
$$\sigma(R^p) = \sqrt{\omega'\Omega\omega} = 0.447214$$

(c)

$$E[R^\star] = R^f + \frac{1}{\sum \omega}E[R^p - R^f]$$
$$= 1.02 + \frac{1}{2}(0.2) = 1.12$$
$$\sigma(R^\star) = \frac{1}{\sum \omega}\sigma(R^p) = 0.223607$$

(d)



2. (a) The first investor would like to invest 0.06/0.09 or 2/3 of his wealth into the first asset. The second investor would like to invest 1/2(0.12)/(0.09) or 2/3 of his wealth into the second asset. Since the first investor has a wealth of 600 and the second one a wealth of 300, aggregate demand for the first asset is 400 and aggregate demand for the second asset is 200. This is exactly equal to aggregate supply, so we are in equilibrium.

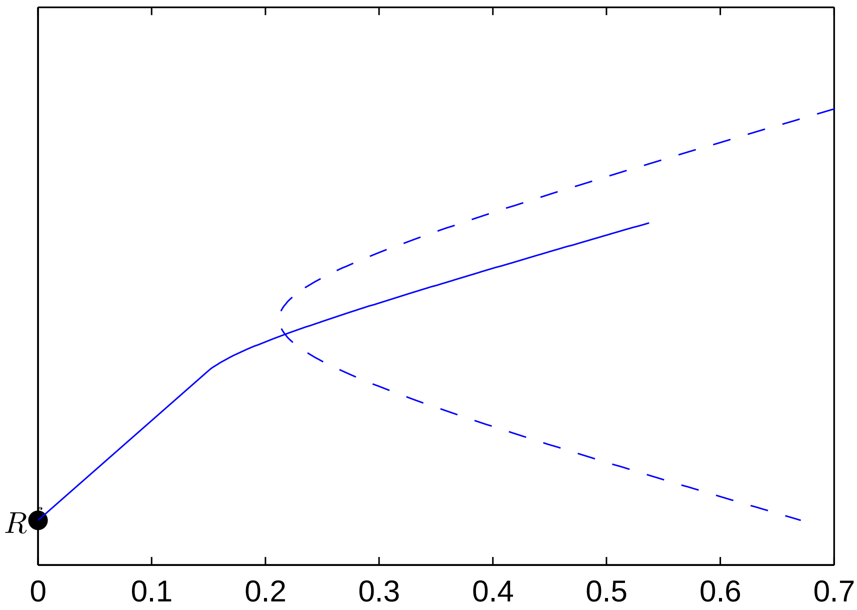
(b) First notice that the weights of the market portfolio is $\omega^m = \begin{bmatrix}2/3 & 1/3\end{bmatrix}$, so the expected excess return of the market portfolio is 0.08. The variance of $R^m - R^f$ and the covariance of the excess returns to the two assets with the excess return of the market portfolio is:

$$\text{var}\left(R^m - R^f\right) = (2/3)^2(0.09) + (1/3)^2(0.09) = 0.05$$
$$\text{cov}\left(R^1 - R^f, R^m - R^f\right) = 2/3(0.09) = 0.06$$
$$\text{cov}\left(R^2 - R^f, R^m - R^f\right) = 1/3(0.09) = 0.03$$

So the beta of the first and second asset is 1.2 and 0.6, respectively. The CAPM predicts that the first asset should earn an expected excess return of 0.096, but it only earns 0.06, while the second asset should earn an expected excess return of only 0.048, but it earns 0.12. The two markets are in equilibrium, but the segmentation of the two markets prevents the equilibrium forces behind the CAPM to come into play.

(c) By the calculations in question (1), we see that the first investor would now demand 400 of the first asset and the 800 of the second one. By a similar calculation we find that the second investor demands 200 of the first asset and 400 of the second one. The aggregate demand (400+100) for the first asset is thus 25 % higher than the supply (400), while the aggregate demand for the second asset (800 + 200) is thus 400 % higher than the supply (200).

(d) At the new expected excess returns, both investors demand 20 % less of the first asset and 80 % less of the second asset (easy to verify), so both markets are now in equilibrium (AD = AS).



3.

(Solid line is linear up to the point 3/4th of the distance from R^f to the tangency portfolio with the RAF, then it bends. The curvy part is constructed by taking 3/4 of the distance between R^f and points on RAF above the tangency point.

4. (a)

$$E[R^{long-short}] = 0.91 - 0.97 = -0.06$$
$$\alpha = 0.52 - (-0.1) = 0.62$$
$$\beta = 0.64 - 1.70 = -1.04$$

(b)

$$E[R^{BAB}] = \frac{1}{0.64}(0.91) - \frac{1}{1.70}(0.97) = 0.85$$
$$\alpha = \frac{1}{0.64}(0.52) - \frac{1}{1.70}(-0.1) = 0.87$$
$$\beta = \frac{1}{0.64}(0.64) - \frac{1}{1.70}(1.70) = 0$$

Notice that the expected return is now very close to the alpha. That's not surprising: You have a beta neutral excess return, so any expected excess return is pure alpha. (Bar rounding errors.)

5. (a)

$$E_t[R^i_{t+1} - X^i_{t+1}] = R^f + \left(\frac{\text{cov}(R^i_{t+1}, R^m_{t+1})}{\sigma^2_m} + \frac{\text{cov}(X^i_{t+1}, X^m_{t+1})}{\sigma^2_m} - \frac{\text{cov}(R^i_{t+1}, X^m_{t+1})}{\sigma^2_m)} - \frac{\text{cov}(X^i_{t+1}, R^m_{t+1})}{\sigma^2_m} \right) \lambda_t$$

(b) i. Investors require a premium to hold stocks that are illiquid at the same time as the market.

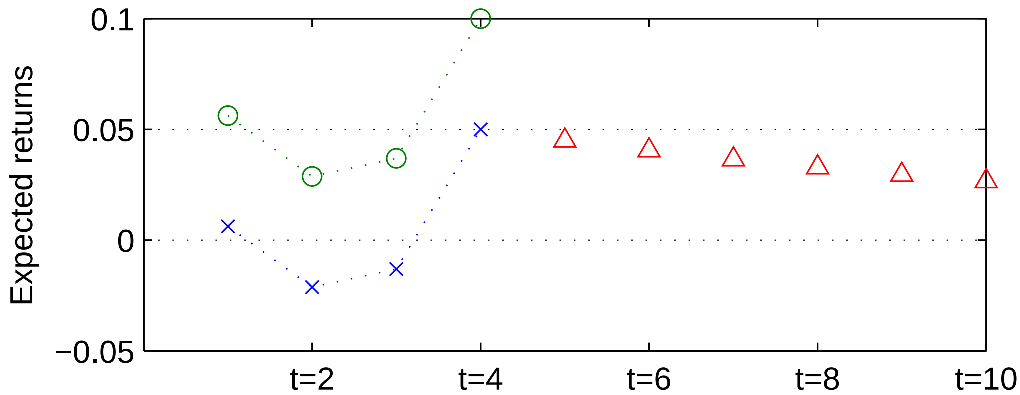
ii. Investors are willing to accept a discount for holding stocks that have high returns when the market is illiquid.

iii. Investors are willing to accept a discount for holding stocks that are liquid in down-markets.

6. As you can see from the figure, up to a year the VR statistic is bigger than 1 and increasing, indicating short term momentum for the aggregate stock market. As we increase the holding period beyond 1 year, the statistic start decreasing again, indicating long term mean reversion.

7. (a) Whenever the required rate of return on an asset goes down, investors bid up the price of the asset until expected returns are as low as the required rate of return; conversely whenever the required rate of return goes down investors want to exit the asset until the price has been driven down enough that the expected return matches the required rate of return again. Hence the negative relationship between the two.

(b)



(c) i.

$$x_t = (0.9)(0.05) - 0.05 = -0.005$$

ii.

$$\begin{aligned} r_{t+1} &= E_t[r_{t+1}] - (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{t+1+j} \\ &= \mu + x_t - \left(\rho + (\rho)^2(\phi) + (\rho)^3(\phi)^2 + \cdots\right)(\epsilon_{t+1}) \\ &= 0.05 + 0.05 - \frac{0.98}{1-(0.98)(0.9)}(-0.05) = 0.5153 \end{aligned}$$

8. (a) i.

$$U'(C_t) = 1/100 \qquad U'(C_{t+1}) = \begin{bmatrix} 1/110 \\ 1/90 \end{bmatrix}$$

ii.

$$m_{t+1} = \theta \frac{U'(C_{t+1})}{U'(C_t)} = \begin{bmatrix} 0.8636 \\ 1.0556 \end{bmatrix}$$

(b) i. $p = E[m_{t+1}x_{t+1}] = (1/2)(0.8636)(2) = 0.8636$, $E[R] = E[x]/p = 1.1579$

ii. $p = E[m_{t+1}x_{t+1}] = (1/2)(1.0556)(2) = 1.0556$, $E[R] = 0.9474$

iii. $p = E[m_{t+1}x_{t+1}] = (1/2)(0.8636)(1) + (1/2)(1.0556)(1) = 0.9596$, $E[R] = 1.0421$

(c) All securities have the same expected payoff of 1, so the first term is the same for all of them. The first security pays of when the discount factor is low, so it's payoffs are negatively correlated with the discount factor, which reduces its price. The second security pays off when the discount factor is high, so it's price is high. The payoff of the third security is constant, so the covariance term is 0, and it's price is just the expected payoff discounted at the risk-free rate.