vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Asset Pricing 4.1
Code:	E_FIN_AP
Coordinator:	Frode Brevik
Date:	October 24, 2013
Time:	08.45–11.30
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question is worth 0.5 points. A total of 10 points can be earned. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than Thursday, November 7, 2012.
Inspection:	Monday, November 15, 2013, 14.15–15:30. Room to be announced.
Number of pages:	8 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her wealth optimally among three stocks, stock 1, 2, and 3, and a risk-free bond. Expected returns are given by

$$R_t^f = 0.99 \qquad E_t[R_{t+1}^e] = \begin{bmatrix} R_{t+1}^1 - R^f \\ R_{t+1}^2 - R^f \\ R_{t+1}^3 - R^f \end{bmatrix} = \begin{bmatrix} 0.03 \\ 0.09 \\ 0.1 \end{bmatrix}$$

Assume that the returns to the first two stocks, R^1 and R^2, both have a standard deviation of $1/4$, while the standard deviation of the return to the 3rd stock, R^3, is $1/3$. The returns to the three stocks are uncorrelated.

The investor chooses a vector of portfolio weights ω for the three stocks to maximize:

$$E[R_{t+1}^p] - \frac{\gamma}{2}\sigma^2(R_{t+1}^p)$$

where $E[R_{t+1}^p]$ and $\sigma^2(R_{t+1}^p)$ are the expected return and the variance of the return to the investors portfolio.

(a) What percentage of her wealth should she allocate to each of the three risky assets if her risk-aversion is $\gamma = 1$?

(b) Compute the expected return and standard deviation of the investor's portfolio.

(c) Compute the expected return and standard deviation of the return to the tangency portfolio between the risky-asset frontier and the mean-variance frontier.

(d) Sketch the Risky-asset frontier and the mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

i. The investor's portfolio.

ii. The tangency portfolio.

iii. Risk-free bonds.

iv. The three stocks.

2. Assume the same setup as in the first question, but now the expected excess returns to the three stocks are the negative of what they were in the first question. In other words:

$$E_t[R_{t+1}^e] = \begin{bmatrix} -0.03 \\ -0.09 \\ -0.1 \end{bmatrix}$$

(a) Find the new optimal portfolio weights of the same investor as above, and also the expected return and standard deviation of his optimal portfolio.

(b) Sketch the new Risky-asset frontier and the new mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

i. The investor's portfolio.

ii. The tangency portfolio.

iii. Risk-free bonds.

iv. The three stocks.

3. The following is an excerpt from table VII from Fama and French 1996.

Table VII

Three-Factor Regressions for Monthly Excess Returns (in Percent) on Equal-Weight NYSE Portfolios Formed on Past Returns: 7/63–12/93, 366 Months

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + e_i$$

The formation of the past-return deciles is described in Table VI. Decile 1 contains the NYSE stocks with the lowest continuously compounded returns during the portfolio-formation period (12-2, 48-2, or 60-13 months before the return month). $t()$ is a regression coefficient divided by its standard error. The regression R^2s are adjusted for degrees of freedom. GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the regression intercepts for a set of ten portfolios are all 0.0. p(GRS) is the p-value of GRS.

	1	2	3	4	5	6	7	8	9	10	GRS	p(GRS)
Portfolio formation months are t-12 to t-2												
a	−1.15	−0.39	−0.21	−0.22	−0.04	−0.05	0.12	0.21	0.33	0.59		
b	1.14	1.06	1.04	1.02	1.02	1.02	1.04	1.03	1.10	1.13		
s	1.35	0.77	0.66	0.59	0.53	0.48	0.47	0.45	0.51	0.68		
h	0.54	0.35	0.35	0.33	0.32	0.30	0.29	0.23	0.23	0.04		
Portfolio formation months are t-60 to t-13												
a	−0.18	−0.16	−0.13	−0.07	0.00	0.02	0.06	0.10	−0.07	−0.12		
b	1.13	1.09	1.07	1.04	0.99	1.00	1.00	1.01	1.06	1.15		
s	1.50	0.83	0.67	0.59	0.47	0.38	0.35	0.40	0.45	0.50		
h	0.87	0.54	0.50	0.42	0.34	0.29	0.23	0.13	−0.00	−0.26		

A typical momentum strategy would be to go long portfolio 10 and short portfolio 1 from the formation period $12 - t$ to $t - 2$. A typical reversal strategy would be to go long portfolio 1 and short portfolio 10 from the based from the formation period $t - 60$ to $t - 13$.

(a) For the formation period $t - 12$ to $t - 2$: Based on the reported alphas (denoted a in the table), what can you say about the overall ability of the Fama-French model to explain momentum returns?

(b) For the formation period $t - 60$ to $t - 13$:

i. Based on the reported alphas, what can you say about the overall ability of the Fama-French model to explain reversal returns?

ii. Based on the reported betas (b, s, and h in the table), which factor(s) if any contribute positively to the returns of the reversal strategy, which factor(s) if any contribute negatively to the returns of the reversal strategy?

4. Carrhart (1997) sort mutual funds into portfolios based on realized past returns.

(a) What would one expect about the average returns of these portfolios if differences in past returns were only due to luck?

(b) Carrhart finds that the return of the portfolio with the best performing funds has a positive beta with the momentum factor, while the return of the portfolio with the worst performing funds has negative beta with the momentum factor. How does Carrhart explain this finding?

5. The table below is taken from Asness et al (2001).

EXHIBIT 4A
MONTHLY REGRESSIONS OF EXCESS HEDGE FUND RETURNS ON CONTEMPORANEOUS AND LAGGED EXCESS S&P 500 RETURNS JANUARY 1994–SEPTEMBER 2000

	Regression Coefficients and t-Statistics						Hypothesis Testing	
Portfolio	Alpha (annualized %)	Beta with S&P 500 (t)	Beta with S&P 500 (t - 1)	Beta with S&P 500 (t - 2)	Beta with S&P 500 (t - 3)	Adjusted R^2	Sum All Betas (= 0)	Sum Lagged Betas (= 0)
Aggregate Hedge Fund Index	-4.45 (-1.16)	0.40 (6.21)	0.12 (1.85)	0.22 (3.37)	0.10 (1.45)	35.3%	0.84 (0.0%)	0.44 (0.1%)
Convertible Arbitrage	-0.98 (-0.46)	0.08 (2.16)	0.16 (4.31)	0.13 (3.46)	0.07 (1.82)	23.8%	0.43 (0.0%)	0.35 (0.0%)
Event-Driven	-2.12 (-0.91)	0.31 (8.04)	0.18 (4.39)	0.08 (1.89)	0.05 (1.19)	47.0%	0.61 (0.0%)	0.30 (0.0%)
Equity Market-Neutral	3.36 (2.32)	0.13 (5.18)	0.05 (1.95)	0.01 (0.39)	0.02 (0.84)	23.4%	0.20 (0.1%)	0.08 (10.8%)
Fixed-Income Arbitrage	-3.78 (-2.08)	0.05 (1.61)	0.10 (3.23)	0.15 (4.84)	0.06 (1.83)	25.2%	0.36 (0.0%)	0.31 (0.0%)
Long/Short Equity	-2.83 (-0.61)	0.57 (7.39)	0.10 (1.25)	0.18 (2.24)	0.14 (1.76)	40.9%	0.99 (0.0%)	0.42 (0.9%)
Emerging Markets	-16.20 (-1.88)	0.79 (5.47)	0.30 (2.02)	0.10 (0.68)	0.06 (0.39)	25.3%	1.25 (0.0%)	0.46 (11.8%)
Global Macro	-6.64 (-1.08)	0.41 (3.94)	0.12 (1.12)	0.37 (3.45)	0.09 (0.83)	21.1%	0.98 (0.0%)	0.57 (0.7%)
Managed Futures	1.72 (0.32)	-0.01 (-0.15)	-0.15 (-1.58)	-0.01 (-0.10)	-0.02 (-0.19)	-1.9%	-0.19 (38.3%)	-0.17 (34.1%)
Dedicated Short Bias	11.59 (2.00)	-1.01 (-10.45)	-0.15 (-1.51)	0.02 (0.22)	-0.13 (-1.26)	57.5%	-1.27 (0.0%)	-0.25 (19.7%)

T-statistics in parentheses. The last two columns report the sum of the contemporaneous and lagged betas (Sum All Betas) and the separate sum of the lagged betas (Sum Lagged Betas); p-values for the F-test versus zero shown in parentheses. Hedge fund and S&P 500 returns used in the regressions are excess of the one-month T-bill return.

Normally, one would only compute the beta with the contemporaneous market return. Why do the authors add betas with the lagged market returns for the hedge fund returns?

6. The table below, taken from Cochrane (2001) gives the empirical relation between the dividend-yield and the future returns and dividend growth for different time periods.

Table 20.1. *OLS regressions of percent excess returns (value weighted NYSE − treasury bill rate) and real dividend growth on the percent VW dividend/price ratio*

Horizon k (years)	$R_{t \to t+k} = a + b(D_t/P_t)$			$D_{t+k}/D_t = a + b(D_t/P_t)$		
	b	$\sigma(b)$	R^2	b	$\sigma(b)$	R^2
1	5.3	(2.0)	0.15	2.0	(1.1)	0.06
2	10	(3.1)	0.23	2.5	(2.1)	0.06
3	15	(4.0)	0.37	2.4	(2.1)	0.06
5	33	(5.8)	0.60	4.7	(2.4)	0.12

$R_{t \to t+k}$ indicates the k-year return. Standard errors in parentheses use GMM to correct for heteroskedasticity and serial correlation. Sample 1947–1996.

According to the Campbell-Shiller decomposition the log dividend yield should be related to future expected returns and dividend growth rates through the identity:

$$d_t - p_t = -\frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j r_{t+1+j}\right] - E_t\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}\right]$$

(a) Comment on the signs of the regression coefficient b in the regressions of future returns and dividend growth on the dividend yield. Are they in line with what you expect from the Campbell-Shiller decomposition?

(b) Assume that log dividend growth is not predictable and has a constant mean of zero and that returns are given by:

$$E_t[r_{t+1}] = 0.05 + x_t$$

where x_t follows the AR(1) process:

$$x_{t+1} = 0.9 x_t + \epsilon_{t+1}.$$

Let $x_t = 0$, $\rho = 0.95$. By how much does the log-dividend yield change in response to the increase in required returns by $\epsilon_{t+1} = 0.1$.

7. An investor maximizing expected utility over wealth at time T can invest in 3 assets:

- A money market account earning a risk-free time-varying continuously compounded risk-free rate of r_t^f.
- Equity earning a constant expected excess return of $\mu_e^x = 0.07$ with a standard deviation of 0.2. Equity returns are uncorrelated with changes to the risk-free rate.
- Long term bonds earning a constant expected excess return of $\mu_b^x = 0.01$ with a standard deviate on of 0.1. The returns to long term bonds have a covariance with changes to the risk-free rate of -0.001.

Stock and bond returns are not correlated. Assume the value function of the investor at time t is given by:

$$V(t, W, r^f) = k(t)e^{(1-\gamma)(T-t)r^f} \frac{W^{1-\gamma}}{1-\gamma}$$

(a) Use the general formula

$$\omega = \left(-\frac{V_W}{V_{WW}W}\right)\Omega^{-1}\mu^x + \left(-\frac{V_{WS}}{V_{WW}W}\right)\Omega^{-1}\Phi$$

to find the optimal allocation to stocks and bonds as a function of the coefficient of relative risk aversion γ and the time left until the final period (T-t).[1]

(b) Assume $T = 20$ and sketch how the optimal allocation to long term bonds evolve with time remaining until T for t between time 0 and 20 for the following two investors:

i. Investor 1 who has a coefficient of relative risk aversion $\gamma = 1$.

ii. Investor 2 who has a coefficient of relative risk aversion $\gamma = 4$.

(time on x-axis, allocation to long term bonds on y-axis.)

(c) Explain economically the difference in how the optimal allocations to long term bonds change with time for the two investors.

[1] Ω denotes the covariance matrix of the returns to stocks and bonds. Φ denotes the vector of covariances between risky-asset returns and changes in the risk-free rate. V_W, V_{WW}, and V_{WS} denote partial derivatives of the value function with respect to wealth and the state.

8. With log-normal consumption growth and a power utility function, the continuously compounded risk-free rate is given by

$$r_t^f = -\log\theta + \gamma E_t[\Delta c_{t+1}] - \frac{1}{2}\gamma^2\sigma_t^2(\Delta c_{t+1})$$

where θ is the time discount factor of the representative investor, γ her coefficient of relative risk aversion and $E_t[\Delta c_{t+1}]$ and $\sigma_t^2(\Delta c_{t+1}$ denote the conditional expectation and conditional variance of log consumption growth between t and $t+1$, respectively.

(a) Give a brief economic motivation why the interest rate should be increasing in expected consumption growth and decreasing in the variance of consumption growth.

Now let log-consumption growth be given by:

$$\Delta c_{t+1} = \mu + x_t + \sigma_c\epsilon_{t+1}$$
$$x_{t+1} = 0.9x_t + \sigma_x\nu_{t+1}$$

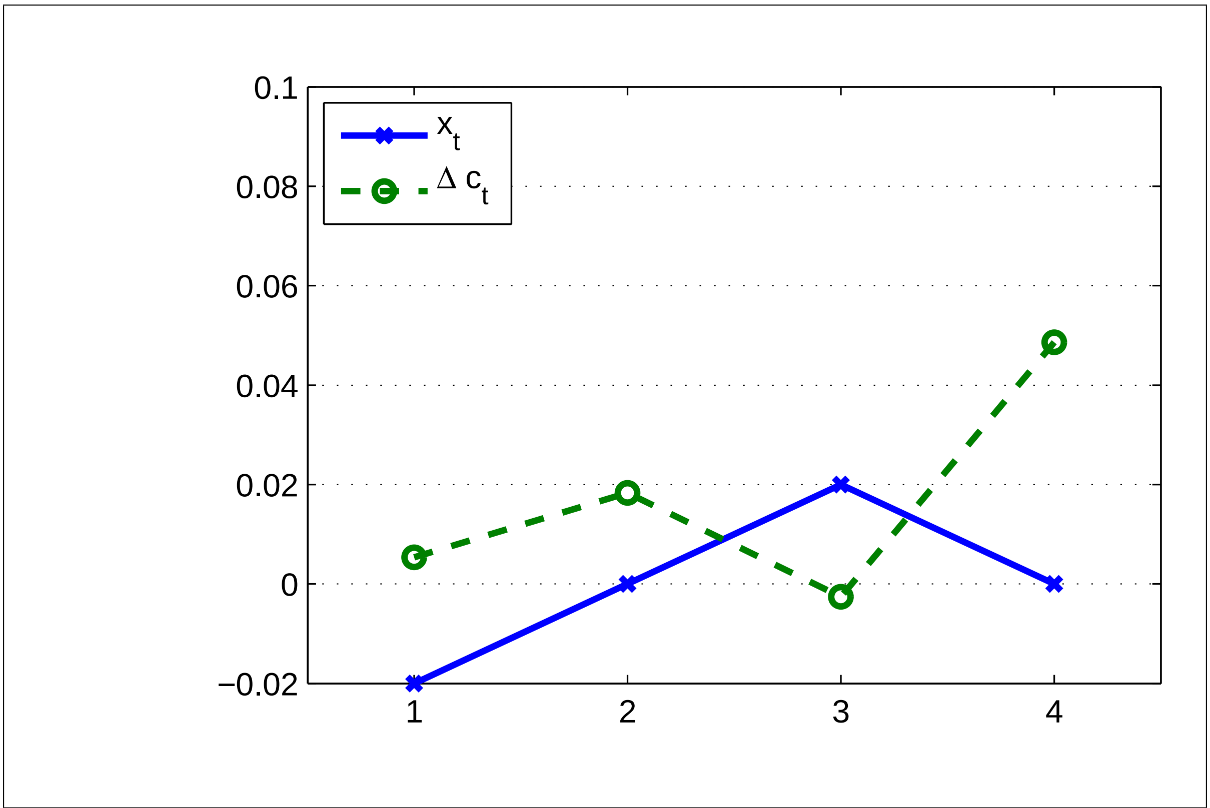
where $\mu = 0.02$, $\sigma_c = 0.01$, $\sigma_x = 0.005$. ϵ and ν are two independent standard normally distributed shocks. x_t is a time-varying component of the trend consumption growth rate that captures business cycle fluctuations in consumption growth.

(b) Assume the representative investor is a power utility maximizer with $\theta = 1$ and $\gamma = 2$. Find an expression for the 1 period continuously compounded risk-free rate as a function of x_t.

(c) i. Find the average risk-free rate. $(E[r^f])$

ii. Find the standard deviation of the risk-free rate $(\sigma(r^f))$.

(d) The following figure contains a random series for x_t and Δc_t.



Copy the figure onto your answer sheet and complete it with the missing series r_t^f.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \qquad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - (E[x])^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

$$x_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

$$E_t[x_{t+k}] = \mu + \theta_k \epsilon_t + \cdots \theta_q \epsilon_{t+k-q} \qquad \text{(conditional expectation)}$$
$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process with zero mean

$$x_t = \phi x_{t-1} + \epsilon_t \qquad \text{with } |\phi| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

$$E[x_t] = 0 \qquad \text{(unconditional expectation)}$$
$$E_t[x_{t+j}] = \phi^j x_t \qquad \text{(conditional expectation)}$$
$$\text{var}\,(x_t) = \frac{1}{1 - \phi^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\phi^j}{1 - \phi^2}\sigma^2 \qquad \text{(auto-covariance)}$$