Solution Key, October 2013

1. (a)

$$\omega = \frac{1}{\gamma}\Omega^{-1}E[R^e] = \frac{1}{1}\begin{bmatrix}16 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 9\end{bmatrix}\begin{bmatrix}0.03\\ 0.09\\ 0.1\end{bmatrix} = \begin{bmatrix}0.48\\ 1.44\\ 0.9\end{bmatrix}$$

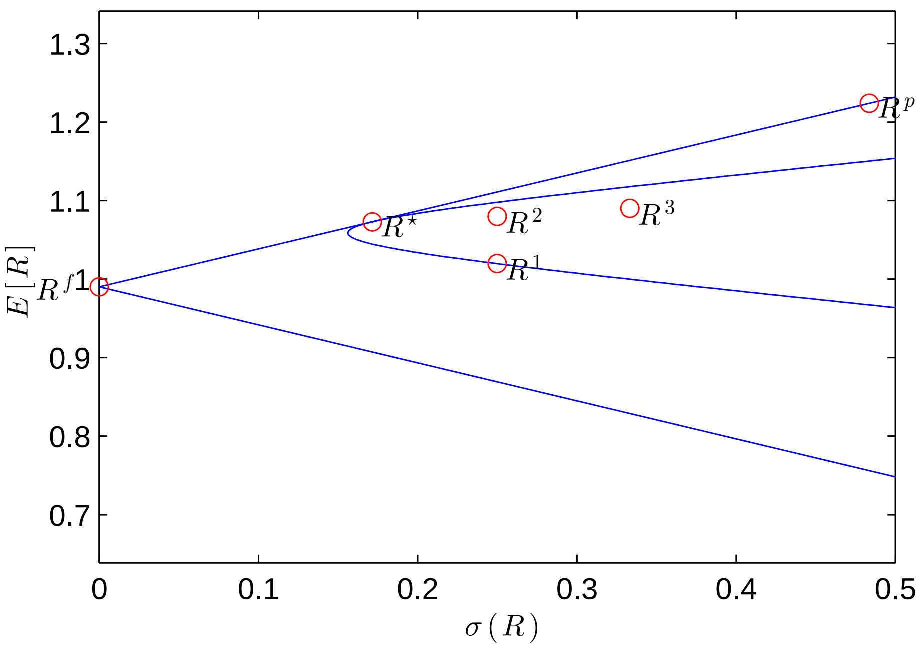
(b)

$$E[R^p] = R^f + \omega' E[R^e] = 1.224$$
$$\sigma(R^p)\sqrt{\omega'\Omega\omega} = 0.483735$$

(c)

$$E[R^\star] = R^f + \frac{1}{\sum \omega}E[R^p - R^f]$$
$$= 0.99 + \frac{1}{2.82}(0.234) = 1.07298$$
$$\sigma(R^\star) = \frac{1}{\sum \omega}\sigma(R^p) = 0.171537$$

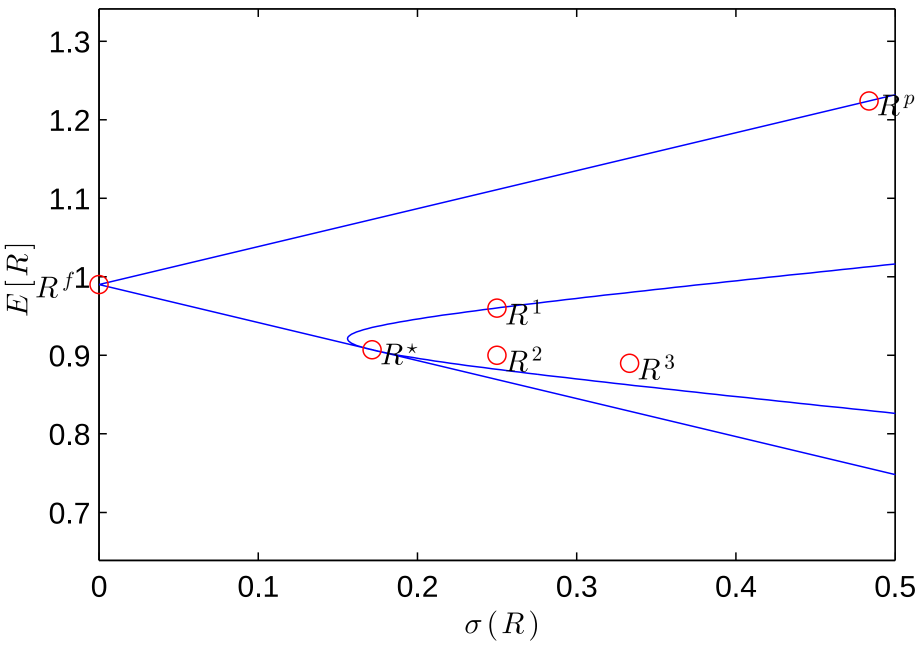
(d)



2. (a) The portfolio weighs are now the negative of what they were before, but the expected return and standard deviation is the same:

$$\omega = \frac{1}{\gamma}\Omega^{-1}E[R^e] = \frac{1}{1}\begin{bmatrix}16 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 9\end{bmatrix}\begin{bmatrix}-0.03\\ -0.09\\ -0.1\end{bmatrix} = \begin{bmatrix}-0.48\\ -1.44\\ -0.9\end{bmatrix}$$
$$E[R^p] = R^f + \omega' E[R^e] = 1.224$$
$$\sigma(R^p)\sqrt{\omega'\Omega\omega} = 0.483735$$

(b)



The only interesting point compared to Q1(d) is that the tangency portfolio is now on lower part of the MVF.

3. (a) It cannot explain momentum returns. There is a clear pattern in the alphas. The portfolio with the lowest past return has a negative alpha of -1.15, while the portfolio with the highest past returns has an alpha of 0.59. The momentum return alpha is about $0.59 - (-1.15) = 1.74$ which is huge (1.74% per month.) Confusion seems to reign here. These kind of patterns means that the model cannot explain the returns. This is the reason Carrhart treats momentum as a 4th factor. If the returns are in line with the FF model they should be small. (Only due to noise.)

   (b) For the formation period $t - 60$ to $t - 13$:

      i. There is nothing left to explain, there is no pattern in the reported alphas and the reversal return would have a total alpha of $-0.18 - (-0.12) = -0.04$, which negative but close to zero. Again, many get it the other way around. If alphas are neglible, the model can explain the returns, not the other way around.

      ii. The returns are basically not related to market beta, but the looser portfolio has a lot more exposure to both the SMB and the HML factor. The SMB and HML betas of the reversal strategy are both roughly 1. $(1.5-0.5)$ and $(0.87-(-0.26))$. According to the Fama-French model it should earn the excess returns to both these factors. It apparently does since there's no detectable alpha left.

4. Carrhart (1997) sort mutual funds into portfolios based on realized past returns.

   (a) They should be similar. The assignment of the mutual funds to portfolios would be completely random, so there's no reason to expect any of the portfolios to be performing better the the others.[1]

   (b) According to Carrhart, this is due to persistence in mutual fund holdings. The mutual funds with the highest past returns tend to have past winner stocks in their portfolios, while those with the lowest past returns tend to have looser stocks in their portfolios. With continuation of past returns (momentum), the best performing mutual funds

[1] Except for the argument of Carrhart below.

will continue to do well for a while, while the worst performing mutual funds will continue to do poorly for a while. (According to Carrhart, it is not due to the best performing funds running momentum strategies. This would be a plausible guess, but wrong according to Carrhart. This is the main point of the paper.)

5. Asness et al (2001) include lagged returns because they are concerned that the reported HF returns are based on stale prices and susceptible to manipulation. (If the returns data were good, these betas would be zero.) This is the main point of the paper. I thought this would be an easy point, but very few of you got it right. Lots of very creative answers here: It has nothing to do with momentum. To check for momentum, you regress on the contemporaneous momentum returns. It has nothing to do with the investment horizon.

6. (a) According to the decomposition we would expect future realized returns to be increasing in the dividend yield (which they are), but realized growth rates to be decreasing in the dividend yield (which they are not). The regression coefficients for the realized returns are as expected, but not those for dividend growth.

   (b) At t, expected returns for all future periods are equal to 0 by the law of iterated expectations:

$$E_t[r_{t+1+j}] = E_t[E_{t+j}[r_{t+1j+j}]] = 0.05 + E_t[x_{t+j}] = 0.05,$$

   After the shock expected returns for returns j periods into the future are given by:

$$E_{t+1}[r_{t+1+1+j}] = 0.05 + E_t[x_{t+1+j}] = 0.05 + (0.9)^j(0.1)$$

   So the total change in the log dividend-yield should be:

$$(0.1) + 0.95(0.9)(0.1) + 0.95^2(0.9)^2(0.1) + \cdots = \frac{1}{1-(0.95)(0.9)}(0.1) = 0.6897$$

   (Which corresponds to a change in the dividend-yield of $e^{0.6897} \approx 2$. Prices would be 50% lower than they would have been without the shock to required rates of return.)

7. (a) Preliminary calculations:

$$\begin{aligned} V_W &= (1-\gamma)\frac{V}{W} \\ V_{WW} &= -\gamma(1-\gamma)\frac{V}{W^2} \\ V_{WS} &= (T-t)(1-\gamma)^2\frac{V}{W} \end{aligned}$$

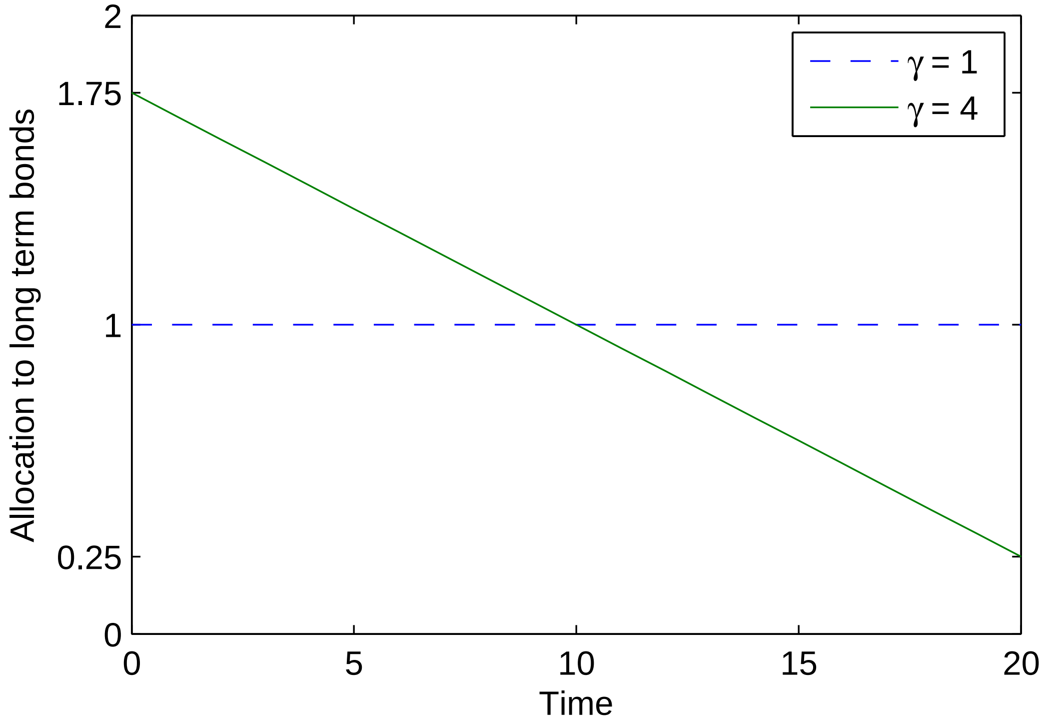
   Which means:

$$\left(-\frac{V_W}{V_{WW}W}\right) = \frac{1}{\gamma}, \qquad \left(-\frac{V_{WS}}{V_{WW}W}\right) = (T-t)\frac{1-\gamma}{\gamma}$$

   Plugging into the general formula, we get the following expression for the optimal portfolio weights:

$$\begin{aligned} \omega &= \frac{1}{\gamma}\begin{bmatrix} 25 & 0 \\ 0 & 100 \end{bmatrix}\begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix} + \frac{1-\gamma}{\gamma}\begin{bmatrix} 25 & 0 \\ 0 & 100 \end{bmatrix}\begin{bmatrix} 0 \\ -0.001 \end{bmatrix}(T-t) \\ &= \frac{1}{\gamma}\begin{bmatrix} 1.25 \\ 1 \end{bmatrix} + \frac{1-\gamma}{\gamma}\begin{bmatrix} 0 \\ -0.1 \end{bmatrix}(T-t) \end{aligned}$$

(b) From the result above, we see that the optimal allocation to risk-free bonds is linear in the time remaining to maturity $(T-t)$, for $\gamma = 1$. The allocation is constant and equal to 1. There is no hedging demand for log utility investors. For $\gamma = 4$, the initial allocation at time $t = 0$ is $1/4 + 3/4(0.1)(20) = 1.75$, and decreases linearly to $1/4$ at $t = T$:



(c) Investor 1 is a mean variance investor, so his allocation to long term bonds only depends on the covariance matrix of the returns. This is because he has $\gamma = 1$, which we know from class means that he has zero hedging demand. His demand is only given by the myopic part. Investor 2 uses long term bonds to hedge against interest rate risk. The hedging demand decreases as time approaches T until there is only the myopic demand left. This makes sense, since the influence of interest rates on his final wealth becomes smaller and smaller. (There is less time remaining for which interest rates can be earned.)

8. (a) Investors want to smooth consumption over time. If they expect consumption growth to be high, they know that marginal utility tomorrow will be lower than today, so they would like to borrow to shift consumption from tomorrow to today. This pushes up the interest rate. If investors face a lot of of uncertainty about their future consumption, they would like to self insure by saving more today, so that they can increase their consumption in bad states of the world. This pushes interest rates down.

(b) Substituting for θ, γ, expected consumption growth $(\mu + x_t)$ and the conditional variance of consumption growth, σ_c^2, in the general expression, we get:

$$\begin{aligned} r_t^f &= 0 + 2(0.02 + x_t) - \frac{1}{2}(2)^2(0.01)^2 \\ &= 0.0398 + 2x_t \end{aligned}$$

(c) i. On average x is 0, so $E[r^f] = 0.0398$

ii. The standard deviation of x_t is $\sqrt{1/(1-(0.9)^2)}(0.005) = 0.0115$, so $\sigma(r^f) = \gamma(0.0115) = 0.023$

(d) (Multiplying the numbers for x_t by 2 and add 0.0398. The numbers for Δc have influence on interest rates. Interest rates are driven by expectations about consumption growth, not realizations.)