vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Asset Pricing 4.1
Code:	E_FIN_AP
Coordinator:	Frode Brevik
Date:	**Trial Exam**
Time:	
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question is worth 0.5 points. A total of 10 points can be earned. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than Thursday, November 8, 2012.
Inspection:	Monday, November 12, 2012, 13.30–15:00. Room to be announced.
Number of pages:	5 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her wealth optimally among three stocks, stock 1, 2, and 3, and a risk-free bond. Expected returns are given by

$$R_t^f = 1.01 \qquad E_t[R_{t+1}] = \begin{bmatrix} R_{t+1}^1 \\ R_{t+1}^2 \\ R_{t+1}^3 \end{bmatrix} = \begin{bmatrix} 1.06 \\ 1.06 \\ 1.034 \end{bmatrix}$$

Assume that the returns to the first two stocks, R^1 and R^2, both have a standard deviation of 1/3, while the standard deviation of the return to the 3rd stock, R^3, is 1/5. The returns to the three stocks are uncorrelated.

The investor chooses a vector of portfolio weights ω for the three stocks to maximize:

$$E[R_{t+1}^p] - \frac{\lambda}{2}\sigma^2(R_{t+1}^p)$$

where $E[R_{t+1}^p]$ and $\sigma^2(R_{t+1}^p)$ are the expected return and the variance of the return to the investors portfolio.

(a) What percentage of her wealth should she allocate to each of the three risky assets if her risk-aversion if her risk aversion is $\lambda = 1$?

(b) Compute the expected return and standard deviation of the investor's portfolio.

(c) Compute the expected return and standard deviation of the return to the tangency portfolio between the risky-asset frontier and the mean-variance frontier.

(d) Sketch the Risky-asset frontier and the mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

i. The investor's portfolio.
ii. The tangency portfolio.
iii. Risk-free bonds.
iv. The three stocks.

2. Assume that there are only two investors in an economy where the assets traded are those of question 1. Each of the investors is a mean variance optimizer with a wealth of 100 and a $\lambda = 1$.[1]

(a) Assume the market capitalization of the first two shares is 90, and that the market capitalization of third share is 120.

i. Would the stock market be in equilibrium? Why/Why not?
ii. Would you expect the CAPM to hold? Why/Why not?

(b) Assume now that one of the two investors is only allowed to hold the first two assets, find her optimal portfolio weights and demand for these two assets.[2]

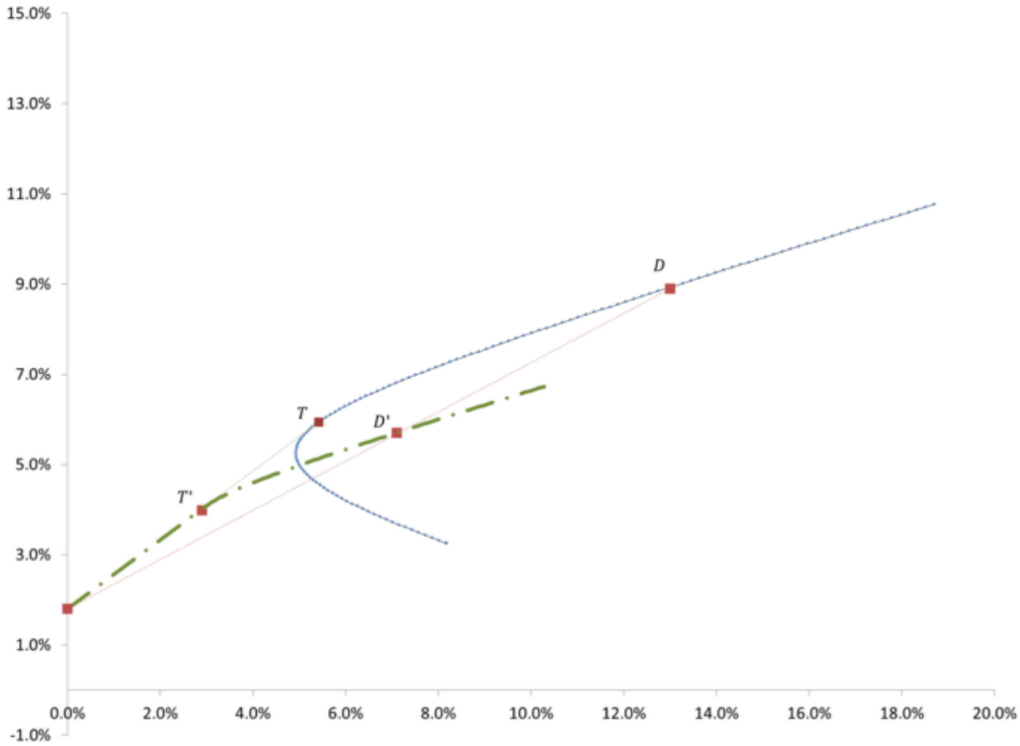
(c) Taking the variances of returns and the market capitalizations from the question above as given, show that if the expected return of the third stock increased to 1.058 the market for this stock would be in equilibrium.

(d) Would the CAPM hold in this new equilibrium? If not, which assets would have expected returns higher than predicted by the CAPM, which assets would have expected returns lower than predicted by the CAPM?

[1] If you were not able to solve question 1, assume the optimal portfolio weights for each investor is $\omega = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}'$

[2] Demand equals portfolio weight times wealth.

3. The figure below is taken from a discussion of Frazzini and Pedersen (2011) from the lecture. The point D' gives the standard deviation and the expected return of the optimal portfolio of an investor who is only allowed to put 60 % of her wealth into risky assets.



Explain why, if enough investors chooses portfolios like this investor, the CAPM will not hold.

4. The Fama-French 3 factor models relates the expected excess return of an asset to exposure to market risk and the SMB and HML factors through

$$E_t[R^i_{t+1} - R^f] = \beta^m_i E_t[R^m_{t+1} - R^f] + \beta^h_i E_t[\text{HML}_{t+1}] + \beta^s_i E_t[\text{SMB}_{t+1}],$$

Assume the expected excess return to the market portfolio is 0.06, the expected return to the SMB factor is 0.04 and the expected return to the HML factor is 0.046.

(a) Find the expected excess return predicted by the Fama-French model to a stock with returns that are uncorrelated with the market return ($\beta^m_i = 0$), and has a beta with both the SMB and HML factor of 1. ($\beta^s_i = \beta^h_i = 1$.)

(b) Suppose the expected return of the stock is actually 0.05. How could you use a position in the stock and the factor portfolios to achieve a higher expected Sharpe ratio than you could have achieved with the factor portfolios alone?

5. Assume the following model for $\widehat{PD}_t$, the deviation of the price-dividend ratio of the stock market from its long term average, and r_{t+1}, the log return on the market portfolio:

$$\begin{aligned} \widehat{PD}_{t+1} &= 0.9\widehat{PD}_t + \epsilon^1_{t+1} \\ r_{t+1} &= 0.06 - 0.002\widehat{PD}_t + \epsilon^2_{t+1}, \end{aligned}$$

where ϵ^1_{t+1} and ϵ^2_{t+1} are independent and have expected value 0.

(a) Find the expected returns $E_t[r_{t+1}]$, $E_t[r_{t+2}]$, and $E_t[r_{t+3}]$ if $\widehat{PD}_t = 0$.

(b) Now assume $\epsilon^1_{t+1} = 5$, so that $\widehat{PD}_{t+1}$ increases to 5.

i. Find the new expectations for the returns at $t + 2$ and $t + 3$. (That is, find $E_{t+1}[r_{t+2}]$ and $E_{t+1}[r_{t+3}]$.)

ii. Explain economically why the expected return for $t + 3$ is different from the expected return for $t + 2$?

6. The current price of a stock is 100, the standard deviation of its annual log return is 50 %, and the continuously compounded risk-free rate is 0.04. In 3 months the company will pay a dividend of 1. For the rest of the exercise, use a binomial tree with nodes 3 months apart that goes from now to 6 months ahead.

(a) Give the numbers for the factors u and d, and the risk-neutral probability $\tilde{\pi}$ of going to the "up-node".

(b) Sketch the tree and fill in stock-prices at every node of the tree.

(c) Use backwards induction to find the current price of an American call option with strike price 100 which expires in 6 months.

7. According to the C-CAPM, the continuously compounded risk-free rate and the Equity Premium should be given by:

$$\begin{aligned} r^f &= -\log\theta + \gamma E_t[\Delta c] - \frac{1}{2}\gamma^2 \text{var}_t\left(\Delta c_{t+1}\right) \\ EP = \mu^m + \frac{1}{2}\sigma^2_m - r^f &= \gamma \text{cov}_t\left(r^m_{t+1}, \Delta c_{t+1}\right) \end{aligned}$$

where μ^m and σ^2_m is the mean and variance of the log return to the market portfolio, r^m_{t+1}, Δc_{t+1} is the log growth rate of aggregate consumption, and θ and γ are the time discount factor and coefficient of relative risk aversion of the average investor, respectively.

(a) Give an economic interpretation of the terms in the expression for the continuously compounded risk-free rate. (Why is the risk-free rate lower, the higher θ is, etc.)

(b) Why does the equity premium depend positively on the covariance of the return to the market portfolio and consumption growth?

(c) Empirically, $\text{cov}_t\left(r^m_{t+1}, \Delta c_{t+1}\right)$ is about 0.0002, the average growth rate of log consumption is about 0.02, and the standard deviation of log consumption growth is about 0.01, and the historical equity premium is about 0.06.

i. What must γ be to match the empirical equity premium?

ii. Assume $\theta = 0.99$. What would be the interest rate be at the γ you found above.

(d) Relate your to the question above to the equity premium puzzle and the risk-free rate puzzle.[3]

[3] If you didn't solve the question, assume you need $\gamma = 50$ to match the historic equity premium and that this corresponds to a continuously compounded risk-free rate of 1.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \qquad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - (E[x])^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

$$x_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process

$$x_t = c + \rho x_{t-1} + \epsilon_t \qquad \text{with } |\rho| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

$$E[x_t] = c/(1 - \rho)$$
$$\text{var}\,(x_t) = \frac{1}{1 - \rho^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\rho^j}{1 - \rho^2}\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \rho^j \qquad \text{(auto-correlation)}$$

VR_k statistic

$$VR_k = \frac{1}{k} \frac{\text{var}\,(r_{t,t+k})}{\text{var}\,(r_{t+1})} = 1 + 2\sum_{j=1}^{k-1} \left(1 - \frac{j}{k}\right) \phi_j,$$

where ϕ_j is the jth autocorrelation coefficient of returns.