Solution key, trial exam

1. (a)

$$\omega = \Omega^{-1}\left[E[R_{t+1}] - R^f\right] = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.05 \\ 0.024 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.45 \\ 0.6 \end{bmatrix}$$

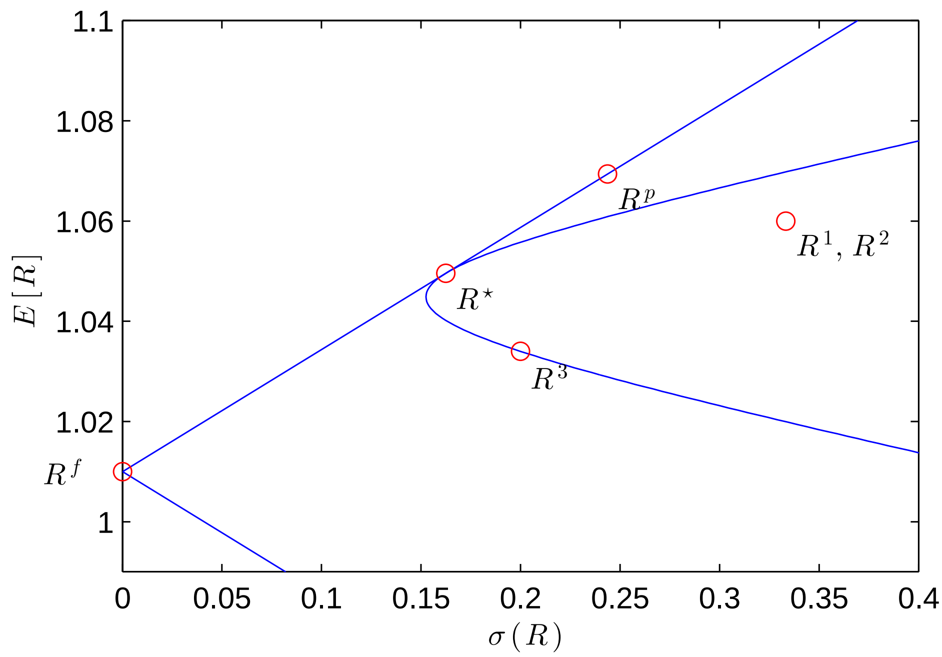
(b)

$$E[R^p] = 1.01 + (0.45)(0.05) + (0.45)(0.05) + (0.6)(0.024) = 1.0694$$
$$\sigma(R^p) = \sqrt{(0.45)^2(1/9) + (0.45)^2(1/9) + (0.6)^2(1/25)} = 0.2437$$

(c) Notice that the investor above allocates 150 % of her wealth to risky assets, so the standard deviation of the return to the portfolio and the expected excess return of her portfolio are both going to be 1.5 times that of the tangency portfolio, so:

$$E[R^\star] = R^f + 2/3(E[R^p] - R^f) = 1.0496$$
$$\sigma(R^p) = (2/3)\sigma(R^p) = 0.1625$$

(d)



2. (a) Yes, since both investors demand 45 of the first 2 shares and 60 of the third share, their aggregate (total) demand would equal aggregate supply.

(b) Yes, the condition needed in the derivation of the CAPM is met. The weights of the market portfolio $\omega^m = \begin{bmatrix} 90/300 & 90/300 & 120/300 \end{bmatrix}' = \begin{bmatrix} 0.3 & 0.3 & 0.4 \end{bmatrix}'$ are exactly those of the tangency portfolio.

(c)

$$\omega = \Omega^{-1}\left[E[R_{t+1}] - R^f\right] = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.45 \end{bmatrix}$$

So the investor will allocate 45 to each of the first two stocks. (Notice that this is the same amount as when she was allowed to allocate to all three assets.

(d) The change in return does not affect the decision of the 1st investor, since she cannot invest in it anyway, but for the second investor, the optimal portfolio weights now change to:

$$\omega = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix} \begin{bmatrix} 0.05 \\ 0.05 \\ 0.048 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.45 \\ 1.2 \end{bmatrix}$$

So, the second investor will still allocate 45 to each of the first stocks, but now allocates 120 to the third stock. Since he is the only one demanding this stock, this is just sufficient to balance demand and supply. (For the other two stocks we are also in equilibrium. Since taken together the two investors still allocate 90 to each.)

(e) No, it wouldn't. Scaling the portfolio weights of the second investor, we see that the weights of the tangency portfolio are $w^\star = \begin{bmatrix} 0.2143 & 0.2143 & 0.5714 \end{bmatrix}'$. These are different from the weights of the market portfolio $w^m = \begin{bmatrix} 0.3 & 0.3 & 0.4 \end{bmatrix}'$, so the CAPM won't hold. The expected returns of the first two assets would be lower than predicted by the CAPM and the expected return to the third asset would be higher than that predicted by the CAPM. To see this, compute the beta's of the each security and the expected excess return of each stock according to the CAPM. (See solutions to week 2 for approach.)

3. The portfolio of this investor overweighs stocks with high expected returns. These are the high beta stocks. If enough investors do this, they will bid up the price of these stocks which will depress the returns of high beta stocks relative to low beta stocks. High beta stocks end up having a lower expected return than predicted by the CAPM and low beta stocks end up having higher expected returns than predicted by the CAPM.

4. (a)

$$E_t[R^i_{t+1} - R^f] = 0.04 + 0.046 = 0.086$$

(b) Start with the tangency portfolio that you get when you only consider the factor portfolios, and then combine it with a small short position in the stock. Since the stock has a negative alpha, you will get a higher Sharpe ratio. (See discussion in notes.)

5. (a) All 0.06

(b) i.

$$E_{t+1}[r_{t+2}] = 0.06 - 0.002(5) = 0.05$$
$$E_{t+1}[r_{t+3}] = 0.06 - 0.002(0.9)(5) = 0.051$$

ii. After the shock at $t+1$, equity is relatively highly valued, which predicts that the expected return for $t+2$ should be relatively low. Since between $t+1$ and $t+2$ valuation is expected to revert back towards its long term average, you expect a lower valuation at $t+2$ and hence a lower expected return for the next period.

6. (a)

$$u = e^{0.5\sqrt{0.25}} = e^{0.25} = 1.284$$
$$d = 1/u = 0.779$$
$$\tilde{\pi} = \frac{e^{0.01} - d}{u - d} = 0.458$$

(b)

$$S^{up}_{t+0.25} = u \cdot 100 = 128.4$$
$$S^{down}_{t+0.25} = d \cdot 100 = 77.9$$
$$S^{up,up}_{t+0.5} = u(128.4 - 1) = 163.63$$
$$S^{up,down}_{t+0.5} = d(128.4 - 1) = 99.2$$
$$S^{down,up}_{t+0.5} = u(77.9 - 1) = 98.7$$
$$S^{down,down}_{t+0.5} = d(77.9 - 1) = 59.9$$

(c) At time $t+0.5$, you exercise if the inner value of the option is positive. Not exercising gives you always a zero payoff, since the option expires at this time:

$$V(t+0.5, S_{t+0.5}) = \max(S_{t+1} - 100, 0) \begin{cases} 63.63 & \text{In up,up state} \\ 0 & \text{In up,down state} \\ 0 & \text{In down,up state} \\ 0 & \text{In down,down state} \end{cases}$$

At time $t+0.25$, the inner value of the option is 28.4 in the up-state, while the discounted expected value in that state is $e^{-0.01}\tilde{\pi}63.63 = 28.81$. You should therefore not exercise, which leaves you with an option value of 28.81. In the down-state, exercising would give you a negative value, while the discounted expected value is 0. The option is therefore worthless in that state.

At time t the discounted value is $e^{-0.01}\tilde{\pi}28.8 = 13.1$. Which is also the option value, since exercising would yield a payoff of 0.

7. (a) See notes week 6.

(b) If the covariance is high, then that means equity tends to high exactly when you need it least, since you then tend to have higher returns exactly when your consumption is high and your marginal utility is low.

(c) Empirically, $\text{cov}_t\left(r^m_{t+1}, \Delta c_{t+1}\right)$ is about 0.0002, the average growth rate of log consumption is about 0.02, and the standard deviation of log consumption growth is about 0.01, and the historical equity premium is about 0.06.

i.
$$\gamma = \frac{0.06}{0.0002} = 300$$

ii.
$$r^f = 0.01 + 300(0.02) - \frac{1}{2}(300)^2(0.01)^2 = 1.51$$

So a continuously compounded risk-free rate of 151%. (That corresponds to an $R^f = 4.53$, for every Euro you put in your bank-account you would have 4.53 at the end of the year.)

(d) The equity premium puzzle is the finding that you can only justify the historical equity premium if you are willing to assume extremely high levels of risk-aversion. In the calculation above 300. Economists tend to think the value should be somewhere between 1 and 5. The risk-free rate puzzle is that, if you use the extreme risk-aversion implied by the equity premium to compute the risk-free rate, you tend to get numbers that are wildly inconsistent with reality.