vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Asset Pricing 4.1
Code:	E_FIN_AP
Coordinator:	Frode Brevik
Date:	October 25, 2012
Time:	08.45–11.30
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question is worth 0.5 points. A total of 10 points can be earned. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than Thursday, November 8, 2012.
Inspection:	Monday, November 12, 2012, 13.30–15:00. Room to be announced.
Number of pages:	5 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her wealth optimally among the stocks of two companies, stock 1, 2, and a risk-free bond. Expected returns are given by

$$R_t^f = 1.02 \qquad E_t[R_{t+1}] = \begin{bmatrix} R_{t+1}^1 \\ R_{t+1}^2 \end{bmatrix} = \begin{bmatrix} 1.10 \\ 1.145 \end{bmatrix}$$

Assume that the returns to stock 1 has a standard deviation of 1/5, while the standard deviation of the return to the 2nd stock is 1/4. The returns to the two stocks are uncorrelated.

The investor chooses a vector of portfolio weights ω for the three stocks to maximize:

$$E[R_{t+1}^p] - \frac{\lambda}{2}\sigma^2(R_{t+1}^p)$$

where $E[R_{t+1}^p]$ and $\sigma^2(R_{t+1}^p)$ are the expected return and the variance of the return to the investors portfolio.

(a) What percentage of her wealth should she allocate to each of the two risky assets if her risk aversion is $\lambda = 2$?

(b) Compute the expected return and standard deviation of the investor's portfolio.

(c) Compute the expected return and standard deviation of the return to the tangency portfolio between the Risky-Asset Frontier and the Mean-Variance Frontier.

(d) Sketch the Risky-asset frontier and the Mean-Variance Frontier in the standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

i. The investor's portfolio.

ii. The tangency portfolio.

iii. Risk-free bonds.

iv. The two stocks.

(e) If the investor chooses the portfolio weights you found in (a), what will be her realized portfolio return R_{t+1}^p if the realized return at $t + 1$ is 1.05 for stock 1 and 1.15 for stock 2?

2. Use the same assets, expected returns and standard deviations as in the first exercise, but assume that there are two investors: The first investor can only take positions in the first asset, while the second can only take positions in the second asset. Both have a wealth of 100 and a risk-aversion $\lambda = 2$. The stock market capitalization of the two shares is both 100.

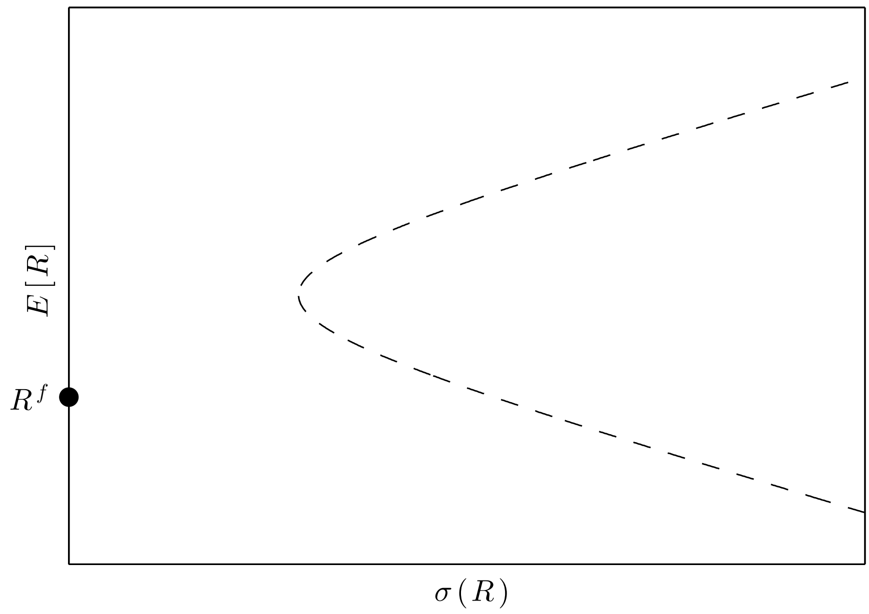
(a) Find the optimal portfolios of each of the investors, and check whether the markets for each of the two stocks are in equilibrium. (Demand = Supply.)

(b) Find the betas of the two assets and show that the CAPM holds.

(c) Now assume that markets are liberalized so that both investors are allowed to take positions in both stocks. Show that a equilibrium obtains if the expected excess returns of both stocks are halved (to 4% and 6.25 %, respectively) with no change in the variances.

(d) Would you expect asset prices to be higher or lower after the liberalization. Argue why.

3. The figure below shows a typical Risky-Asset Frontier together with the risk-free rate:



Copy the figure to your solution sheet, and, in the same figure, draw the upper part of the Mean-Variance Frontier for an Investor who is not allowed to use leverage. (So his allocation to risky-assets smaller or equal to 100 % of wealth.)

4. The Fama-French 3 factor models relates the expected excess return of an asset to exposure to market risk and the SMB and HML factors through

$$E_t[R^i_{t+1} - R^f] = \beta^m_i E_t[R^m_{t+1} - R^f] + \beta^h_i E_t[\mathrm{HML}_{t+1}] + \beta^s_i E_t[\mathrm{SMB}_{t+1}],$$

Assume the expected excess return to the market portfolio is 6 %, the expected return to the HML factor is 4.5 % and the expected return to the SMB factor is 4 %.

(a) Describe how Fama-French constructed the SMB factor.

(b) Suppose a stock has a beta both with the market and the SMB factor of 0.5 ($\beta^m_i = \beta^s_i = 0.5$), but its returns is uncorrelated with the return to the HML factor. Find the expected excess return of the stock according to the Fama-French model.

(c) Suppose the expected excess return to the stock is actually 6 %. How could you use a position in the stock and the factor portfolios to achieve a higher expected Sharpe ratio than you could have achieved with the factor portfolios alone?

5. Assume the following model for $\widehat{dp}_t$, the deviation of the log dividend-yield of the stock market from its long term average, and r_{t+1}, the log return on the market portfolio:

$$\widehat{dp}_{t+1} = 0.8\widehat{dp}_t + \epsilon^1_{t+1}$$
$$r_{t+1} = 0.06 + 0.06\widehat{dp}_t + \epsilon^2_{t+1},$$

where the shocks ϵ^1_{t+1} and ϵ^2_{t+1} both have expected value 0.

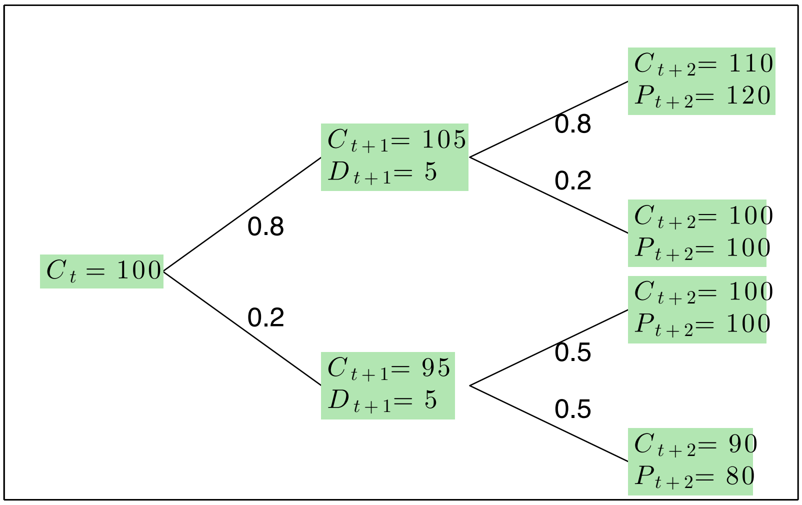
(a) Find $\underline{\widehat{dp}}$, the level of $\widehat{dp}_t$ at which the expected log return for $t+1$ is exactly 0.

(b) Assuming $\widehat{dp}_t = \underline{\widehat{dp}}$, find the expected return at time t for $t+1$, $t+2$ and $t+3$.[1]

6. Consider the following simple economy. The probabilities of going up or down depend on the state of the economy. In the upstate at time $t+1$, the probabilities are 0.8 and 0.2, the same as at time t, while in the downstate, both probabilities are 0.5. The representative (typical) investor has a utility function given by

$$U(C) = \frac{1}{2}\sqrt{C},$$

and he has a time discount factor of $\theta = 0.99$.



(a) Assume that the values for C_t and C_{t+1} given in the figure are the equilibrium consumption levels of the representative investor. Find the implied values of the stochastic discount factor between time t and $t+1$, and times $t+1$ and $t+2$ for all possible states of the economy.

(b) The numbers given for P_{t+2} are the prices of a particular stock in different states of the economy at time $t+2$. This stock pays a dividend of 5 at time $t+1$, but no dividend at $t+2$. Find the ex-dividend price of the stock at time in both states at time $t+1$ and at time t.

(c) Find the expected return of the stock at t and in both states at $t+1$.

(d) Find the risk-free rate R^f_t at t and in both states at time $t+1$.

(e) How would the representative investor like to change his consumption in the down state at $t+1$ if the risk-free rate would be the same as you found for the up-state at $t+1$. Why wouldn't that be an equilibrium?

[1]That is, find $E_t[r_{t+1}]$, $E_t[r_{t+2}]$, and $E_t[r_{t+3}]$.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \qquad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - (E[x])^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$e\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

$$x_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process

$$x_t = c + \rho x_{t-1} + \epsilon_t \qquad \text{with } |\rho| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

$$E[x_t] = c/(1 - \rho)$$
$$\text{var}\,(x_t) = \frac{1}{1 - \rho^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\rho^j}{1 - \rho^2}\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \rho^j \qquad \text{(auto-correlation)}$$

VR_k statistic

$$VR_k = \frac{1}{k}\frac{\text{var}\,(r_{t,t+k})}{\text{var}\,(r_{t+1})} = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\phi_j,$$

where ϕ_j is the jth autocorrelation coefficient of returns.