vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Asset Pricing 4.1
Code:	E-FIN-AP
Coordinator:	Frode Brevik
Date:	December 12, 2011
Time:	8:45
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question is worth 0.5 points. A total of 10 points can be earned. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than Monday, December 19, 2011.
Inspection:	Monday, December 19, 2011 13.30–15:00. Room to be announced.
Number of pages:	5 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her wealth optimally among three risky assets and a risk-free bond. The risk-free rate and the expected returns to the three risky assets are given by

$$R^f = 1.02 \qquad E_t[R_{t+1}] = \begin{bmatrix} 1.06 \\ 1.07 \\ 1.12 \end{bmatrix}$$

The returns to the three risky assets are uncorrelated. The standard deviation of the return to the first risky asset is 20% while the returns to the two other risky assets have a standard deviation of 10%. The investor chooses a vector of portfolio weights w for the two stocks to maximize:

$$E[R^p_{t+1}] - \frac{\lambda}{2}\sigma^2(R^p_{t+1})$$

where $E[R^p_{t+1}]$ and $\sigma^2(R^p_{t+1})$ are the expected return and the variance of the return to the investors portfolio.

   (a) What percentage of her wealth should an investor with a risk-aversion parameter $\lambda = 8$ allocate to each of the three risky assets and to risk-free bonds.

   (b) Compute the expected return and standard deviation of the investor's portfolio.

   (c) Compute the expected return and standard deviation of the return to the tangency portfolio between the risky-asset frontier and the mean-variance frontier.

   (d) Find the maximum Sharpe ratio that it is possible to achieve by combining the three risky assets available to the investor.

   (e) Sketch the Risky-asset frontier and the mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

      i. The Investor's portfolio.
      ii. The tangency portfolio.
      iii. Risk-free bonds.
      iv. The theee risky assets.

   (f) The first risky asset available to the investor is actually the market portfolio, while the second and third risky asset are factor portfolios capturing a value factor and a momentum factor, respectively. Assuming all other parameters stay the same, what should the expected returns to these two factors have been if the CAPM was true.

2. An investor's utility of wealth is given by

$$U(W) = -e^{-W}$$

Show that the investor's preferences exhibit constant absolute risk aversion and increasing relative risk aversion.

3. A crucial component of any model of financial markets is a specification of how agents form expectations. Describe briefly (ideally with an example) the following biases document by psychologist for how people appear to form beliefs in practice:

   - Overconfidence.
   - Anchoring.

4. (Liquidity spirals) Explain how initials losses of speculators can set off a liquidity spiral that drives prices away from fundamentals.

5. (Binomial tree option pricing) The current price of a stock is 100 and the standard deviation of its annual log return is 0.5. The stock pays no dividends. The continuously compounded annual risk-free rate is $r^f = 0.04$. For the rest of the exercise, use a binomial tree with nodes 3 months apart that goes from the current period to 6 months into the future. .

(a) Give the numbers for the factors u and d, and the risk-neutral probability $\tilde{\pi}$ of going to the "up-node".

(b) Use backwards induction to price at each node a European put option with strike price of 100 which matures in 6 months.

6. (Campbell-Shiller) Ignoring a constant term, the dynamic accounting identity of Campbell-Shiller studied in class shows that the log-dividend yield of the market portfolio is approximately related to expected future dividend growth and returns through:

$$d_t - p_t = E_t \left[\sum_{j=0}^{\infty} \rho^j \big(- \Delta d_{t+1+j} + r_{t+1+j} \big) \right] \tag{1}$$

where d_t, p_t are the log dividends and the log price at time t, respectively, Δd_{t+1+j} and r_{t+1+j} are the growth rates of log dividends and the log return at time $t+1+j$, respectively, and ρ is positive number smaller than 1.

(a) Explain economically why the log-dividend yield is negatively related to expected future dividend growth and positively related to expected future returns.

(b) If log dividends follow a random walk with a drift (so that expected future log dividend growth is constant), what would a high log-dividend yield imply for future expected returns?

7. Assume that interest rates are time-varying, but expected excess log returns to all risky assets are constant over time. An investor can invest in two risky assets: equity and gold. The log return to equity is denoted by r^{equity}, while the log return to holding gold is denoted by r^{gold}. All log returns are conditionally normal with a constant covariance matrix Ω. The investment opportunity set, as a function of the continuously compounded interest rate r_t^f is given by:

$$\mu = \begin{bmatrix} E[r_{t+1}^{equity}] + \sigma^2(r_{t+1}^{equity}) \\ E[r_{t+1}^{gold}] + \sigma^2(r_{t+1}^{gold}) \end{bmatrix} = r_t^f + \begin{bmatrix} 0.05 \\ -0.01 \end{bmatrix} \quad \Omega = \begin{bmatrix} (0.2)^2 & 0 \\ 0 & (0.3)^2 \end{bmatrix}$$

The current risk free rate is given by

$$r_t^f = 0.01$$

The realized return to equity is uncorrelated with changes in the interest rate, while the holding return to gold has a negative covariance with changes to the risk-free rate of -0.001.

(a) Assume an investor has a time invariant value function given by:

$$V(W_t, r_t^f) = k\ r_t^f + \ln W_t,$$

where k is a constant. Find the investors optimal portfolio using the formula[1]

$$w = \left(- \frac{V_w}{V_{ww} W} \right) \Omega^{-1} (\mu - r_f) + \left(- \frac{V_{ws}}{V_{ww} W} \right) \Omega^{-1} \Phi$$

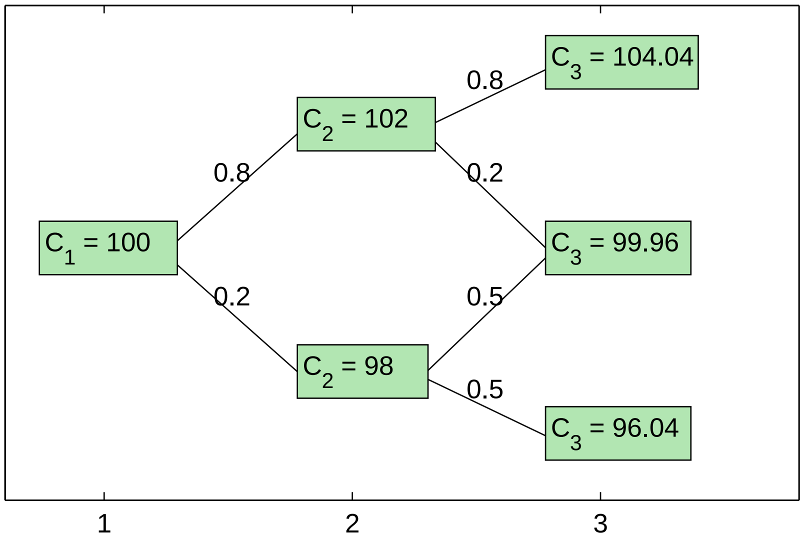
Give your answer in terms of myopic demand and hedging demand.

(b) How would you expect the hedging demands of investors who are more and less risk averse than this investor to differ. Motivate your answer economically.

(c) Given that the supply of gold is positive, what does your answer to question (b) tell you about what the risk aversion of the representative (typical) investor in the economy is?

[1] Φ denotes the vector of covariances of risky-asset returns and changes to the state variable r^f, V_w and V_{ww} denote the first and second derivative of the value function with respect to wealth, respectively and V_{ws} denote the cross-derivative of the value function with respect to wealth and the state r_t^f.

8. Consider the following simple economy. The probability of going up is equal to 0.8 initially, but switches to 0.5 if we go down once. C_t is consumption in year t depending on the state of the economy.



(a) Assume the representative investor has a period utility function given by

$$U(C) = \ln C$$

And a time discount factor of $\theta = 0.99$. Find the discount factor m_{t+1} at each node in the two years $t = 2$ and $t = 3$.

(b) Use the discount factor you just found together with the probabilities on the tree to find:

i. The prices of a 1 period discount bonds in the up and down nodes at time $t = 2$, as well as in the initial node.

ii. The price of a 2 period discount bond at time $t = 1$.

(c) Explain economically why the price of 1 period bonds differ in the way they do in the up and down nodes at time $t = 2$.

(d) Find the yield on 1 and 2 period bonds at time $t = 1$ and use your result to sketch the initial term structure of interest rates (yield curve).

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \qquad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - (E[x])^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

$$x_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process

$$x_t = c + \rho x_{t-1} + \epsilon_t \qquad \text{with } |\rho| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

$$E[x_t] = c/(1 - \rho)$$
$$\text{var}\,(x_t) = \frac{1}{1 - \rho^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\rho^j}{1 - \rho^2}\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \rho^j \qquad \text{(auto-correlation)}$$

VR_k statistic

$$VR_k = \frac{1}{k}\frac{\text{var}\,(r_{t,t+k})}{\text{var}\,(r_{t+1})} = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\phi_j,$$

where ϕ_j is the jth autocorrelation coefficient of returns.