Solution Key: December 2011

1. (a) From

$$w = \frac{1}{\lambda}\Omega^{-1}(\mathrm{E}[R] - R^f \iota) = \frac{1}{8}\begin{bmatrix} 25 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}\begin{bmatrix} 0.04 \\ 0.05 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 1/8 \\ 5/8 \\ 10/8 \end{bmatrix}$$

we see that the investor should allocate 12.5 % of his wealth to asset 1, 62.5 % of his wealth to asset 2, 125 % of his wealth to asset 3, and take a short position equal to 100 % of his wealth in the risk-free asset.

(b)

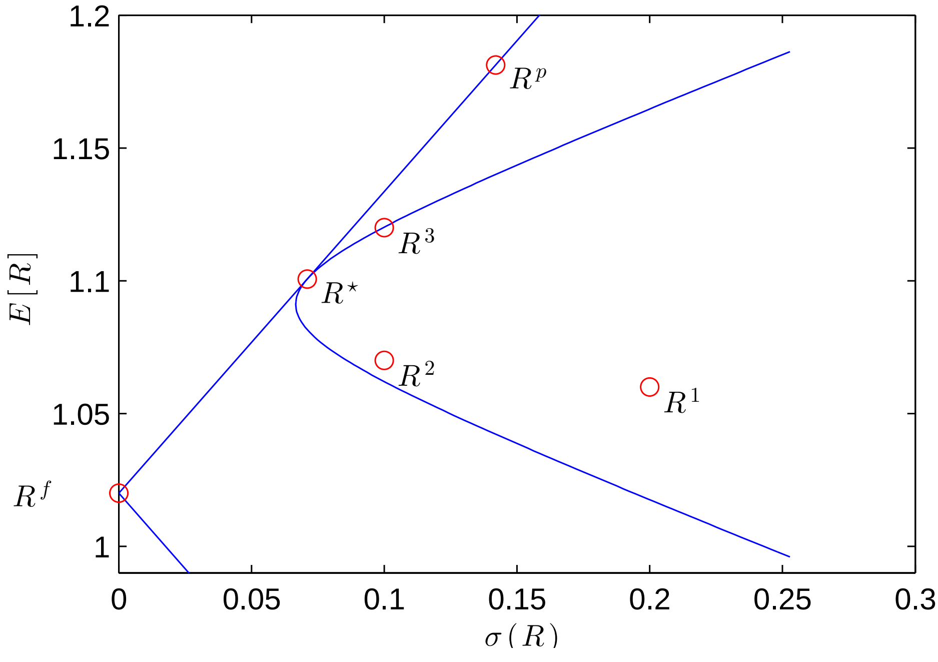
$$E[R^p] = 1.02 + (1/8)(0.04) + (5/8)(0.05) + (10/8)(0.1) = 1.1813$$
$$\sigma(R^p) = \sqrt{w'\Omega w} = 0.1420$$

(c)

$$E[R^\star] = 1.1006$$
$$\sigma(R^\star) = 0.071$$

(d) (Using equality of slope of MVF and maximum Sharpe ratio.)

$$SR^{\max} = \frac{E[R^\star] - R^f}{\sigma(R^\star)} = \frac{0.0806}{0.071} = 1.1358$$



(e)

(f) For the CAPM to hold, we would need the expected excess returns to the two other assets to be zero, since their returns are uncorrelated with the returns to the market portfolio.[1]

2. Since $U'(W) = e^{-W}$ and $U''(W) = -e^{-W}$, we have that

$$R_A(W) = -\frac{U''(W)}{U'(W)} = 1$$
$$R_R(W) = -\frac{U''(W)W}{U'(W)} = W$$

[1] There are two ways to understand this: The deep one is to notice that as long as their expected excess returns are positive investors mean-variance would like to overweigh them in their portfolios. Alternatively, notice that the beta of both assets is zero, since the returns are uncorrelated, so their expected asset returns need to be zero according to the CAPM.

So the investor's absolute risk aversion is constant and her relative risk aversion is increasing.

3. - Overconfidence: People tend to have excess trust in their judgements. Experiments such as the one we did in class show that the confidence intervals people assign to their estimates of quantities are too narrow. E.g. a 90 % confidence interval would cover the true value only 40 % of the time. (Possible examples from investments include the beliefs that house prices can only go up or that sovereign bonds will never default, or that most investors seem to think they can beat the market.)
   - Anchoring: When forming estimates people tend to start with some initial, possibly arbitrary value, and then adjust away from it. A classic example from psychology is the wheel of fortune experiment, where the experimenter has the subject watch him spin a wheel of fortune before asking him a question like the number of African nations in the UN. Subjects tend to give answers close to random number generated by the wheel of fortune. (An example from investments would be that people tend to attach to more probability to a stock going up 10 percent if the price was 10 % higher a year ago then when the price was 10 % lower a year ago. I.e. they anchor their expectations for future prices to past prices.)

4. An initial loss on a speculative position might set off margin calls which require speculators to reduce their speculative positions. This reduction in demand might set off a price movement in the opposite direction of what the speculator is betting on. (The price might move against fundamentals.) This would lead to further losses on the speculative position and might trigger the funding institution to increase the margin requirements. In response to higher margin requirements, speculators need to further reduce their speculative positions, etc.

5. (Binomial tree option pricing)

   (a)

$$\begin{aligned} u &= e^{0.5\sqrt{0.25}} = e^{0.25} = 1.284 \\ d &= 1/u = 0.779 \\ \tilde{\pi} &= \frac{e^{0.01} - d}{u - d} = 0.458 \end{aligned}$$

   (b) After 6 months, the option is only in the Money at the lowest node, where it's worth 39.35. Applying backwards induction we find that it's worth the 0 in the up-node at 3 months, and $e^{-0.01}(1 - 0.458)(39.35) = 21.125$. Finally, we find that the current value of the option is $e^{-0.01}(1-0.458)(21.125) = 11.34$. (Most of the students forget to discount the expected value of the option at $t + \Delta t$, some even compound the price (multiply with $e^{0.01}$ instead of $e^{-0.01}$.) Risk neutral valuations compels you to discount future values at the risk-free rate.)

6. (a) The higher the expected future dividend growth, the higher the present value of the expected dividend stream and so the the current price, which lowers the current dividend yield. For given expectations about cash-flows, the only way expected returns can be higher is if the current price is low.

   (b) In this case any change in the dividend yield translates directly into changes in expected returns. (By constructions expected dividend growth rates are constant, so none of the variation in dividend yields can be explained by changes in expected future dividend growth rates.) A high dividend yield must translate into high expected future returns.

7. (a) Since we have $V_w = 1/W$, $V_{ww} = -1/W^2$, and $V_{ws} = 0$, we find the myopic demand of the investor is:

$$w^{myopic} = \left(\frac{1/W}{(1/W^2)W}\right) \begin{bmatrix} 25 & 0 \\ 0 & 100/9 \end{bmatrix} \begin{bmatrix} 0.05 \\ -0.01 \end{bmatrix} = \begin{bmatrix} 1.25 \\ -0.11 \end{bmatrix},$$

   whereas the hedging demand is zero.

   (b) An investor with a higher coefficient of relative risk aversion than this investor would care more about the downside and make sure he has sufficient funds available when expected returns are low. Because of the negative covariance between returns to holding gold and changes in the risk-free rate, a long position in gold would be a good hedge against worse investment opportunities. An investor with a lower coefficient of relative risk aversion would care more about the upside of having extra wealth when investment opportunities are good. This is achieved by a short position in gold.

(c) In equilibrium, we need demand to equal supply. Since the supply is positive, we need the typical investor to have a positive demand for gold. (Unlike the investor in question (a).) From (b), we conclude that for the representative investor to want to have a long position in gold, we need him to have a coefficient of relative risk aversion higher than the investor in the example. (I.e. higher than 1).

8. (a) In all up-nodes $m_{t+1} = 0.99(1/1.02) = 0.9706$, in all the down-nodes $m_{t+1} = 0.99(1/0.98) = 1.0102$

(b) i. In the up-node and the initial node: $B^{(1)} = (0.8)(0.9706) + (0.2)(1.012) = 0.9785$. In the down node: $B^{(1)} = (0.5)(0.9706) + (0.5)(1.012) = 0.9904$. Initial node

ii. $B^{(2)} = (0.8)(0.9706)(0.9785) + (0.2)(1.012)(0.9904) = 0.9599$

(c) Expected consumption growth is higher in the up-state, which makes it less interesting to save. Investors are only willing to hold bonds if their prices are lower. (Equally valid explanations would involve expected marginal utility or differences in the expected value of the discount factor).

(d) The yield on the 1 year bond is: $1/0.9789 = 1.0219$, the yield on the two year bond is $1/\sqrt{0.9599} = 1.0207$, so the term structure is slightly down-ward sloping.