vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Asset Pricing 4.1
Code:	60412040
Coordinator:	Frode Brevik
Date:	October 24, 2011
Time:	8:45
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question is worth 0.5 points. A total of 10 points can be earned. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than Monday, October 31, 2011.
Inspection:	Monday, October 31, 2011 13.30–15:00. Room to be announced.
Number of pages:	6 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her wealth optimally among two stocks and a risk-free bond. The returns to the three stocks are jointly normally distributed with $R_{t+1} \sim \mathcal{N}(\mu, \Omega)$. Expected returns are given by

$$R^f = 1.01 \qquad E_t[R_{t+1}] = \begin{bmatrix} 1.07 \\ 1.05 \end{bmatrix}$$

The return of each risky asset has a standard deviation of 1/3 and the correlation coefficient between the returns to the two risky assets is equal to 1/2. The investor chooses a vector of portfolio weights w for the two stocks to maximize:

$$E[R^p_{t+1}] - \frac{\lambda}{2}\sigma^2(R^p_{t+1})$$

where $E[R^p_{t+1}]$ and $\sigma^2(R^p_{t+1})$ are the expected return and the variance of the return to the investors portfolio.

   (a) What percentage of her wealth should an investor with a risk-aversion parameter $\lambda = 6/5$ allocate to each of the two risky assets?
   (b) Compute the expected return and standard deviation of the investor's portfolio.
   (c) Compute the expected return and standard deviation of the return to the tangency portfolio between the risky-asset frontier and the mean-variance frontier.
   (d) Sketch the Risky-asset frontier and the mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:
      i. The Investor's portfolio.
      ii. The tangency portfolio.
      iii. Risk-free bonds.
      iv. The two stocks.
   (e) Assume that the two stocks are the only risky assets traded in the economy. What would the relative market capitalization of the two companies need to look like for the CAPM to hold?

2. The figure below shows a time series percentage of historical deviations from theoretical parity prices between shares for Royal Dutch and Shell.[1]

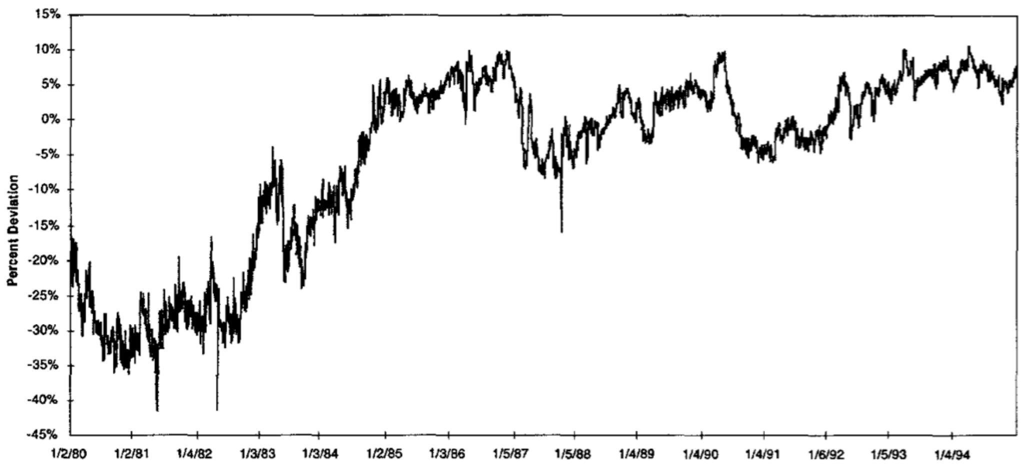


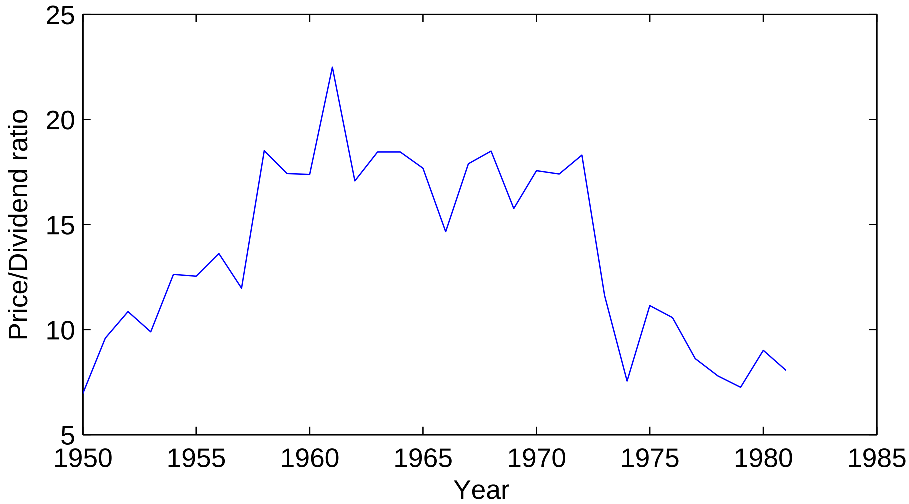
Fig. 1. Log deviations from Royal Dutch/Shell parity. Source: Froot and Dabora (1999).

Relate what you see to the limits of arbitrage for short term investors.

3. Describe how Fama and French construct their value factor (HML).

[1] Royal Dutch was traded in Amsterdam and New York, Shell was traded in London.

4. The following figure shows the evolution of the price-dividend ratio of the portfolio of stocks in the S&P 500 index between 1950 and 1981.



How does Robert Shiller in his 1984 article explain the rise in the valuation of the stock market during the 1950s and its decline in the 1970s?

5. Assume log-returns on the market portfolio are generated by

$$r_{t+1} = \mu + \sigma \epsilon_{t+1} \qquad \epsilon_{t+1} \sim I.I.D.N(0,1)$$

and that the continuously compounded risk-free rate r^f is constant.

(a) Let $r_{t,t+k}$ denote the log return over k periods from time t to $t+k$. ($r_{t,t+k} = r_{t+1} + r_{t+2} + \ldots + r_{t+k}$). Describe how

i. $E_t[r_{t,t+k}]$,

ii. $\sigma^2(r_{t,t+k})$,

iii. $SR_k = \frac{E_t[r_{t,t+k} - kr^f]}{\sigma(r_{t,t+k})}$

changes with the number of periods k.

(b) A financial analyst has looked at the realized returns of the market portfolio and finds that the the expected return grows linearly with the holding period, but that the standard deviation of realized returns grows only with the square root of the holding period. Does this mean that equity is a less risky investment for longer horizons? Please provide an economic argument.

(c) Investors A and B both allocate 50 % of their initial wealth of 100 Euros to the market portfolio at time t and the remainder to risk-free bonds. Investor A is a buy-and-hold investor and does not trade anything at time $t+1$. Investor B rebalances her portfolio at time $t+1$ so that she maintains a fixed weight of 50 % allocated to the market portfolio portfolio. The realized gross return to the market portfolio is 0.5 at time $t+1$ and 2 at time $t+2$.[2] The gross risk-free rate is constant at $R^f = 1$. Find the wealth of both investors at t+2.

6. Campbell and Shiller decompose unexpected stock returns into

$$r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t)\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{t+1+j}$$

Assume $\rho = 0.95$. Let log dividend growth be given by

$$\Delta d_{t+1} = 0.5 \Delta d_t + \epsilon_{t+1}$$

(a) Sketch how expected log dividend growth rates (Δd_{t+j}) change from time t to $t+1$ in response to $\epsilon_{t+1} = 0.1$ for $j = \{1, 2, 3, 4\}$

[2]The market goes down by 50 % in t+1 and up by 100 % at t+2.

(b) Compute the effect of this news about future dividend growth on r_{t+1}.

(c) Assume that expected future returns are constant, so that

$$E_{t+1}[r_{t+1+j}] = E_t[r_{t+1+j}], \quad j \geq 1$$

State for $k = 1, \ldots, 3$, the implied value of the variance-ratio statistic:

$$VR_k = \frac{\sigma^2(r_{t,t+k})}{k\sigma^2(r_{t+1})}$$

7. Acharaya-Pedersen (2005) assume that every investor is a mean-variance optimizer with an investment horizon of 1 period and show that, in equilibrium, there is a CAPM relation for net returns:

$$R^{i,net}_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - X^i_{t+1}$$

where X^i_{t+1} is the stochastic transaction cost for trading stock i at $t+1$, measured as a percentage of the price at time t. The equilibrium requires:

$$E_t[R^i_{t+1} - X^i_{t+1}] = R^f + \frac{\text{cov}(R^i_{t+1} - X_{t+1}, R^m_{t+1} - X^m_{t+1})}{\text{var}(R^m_{t+1} - X^m_{t+1})}\lambda_t$$

where

$$\lambda_t = E_t\left[R^m_{t+1} - X^m_{t+1} - R^f\right]$$

and R^m_{t+1} and X^m_{t+1} is the realized return to the market portfolio and the transaction cost for trading the market portfolio, respectively:

(a) Decompose this relation to show that the expected return depends on the covariance of the asset returns with the return to the market portfolio and 3 liquidity risks.

(b) Give a short economic interpretation for each of the 3 liquidity risks.

8. Assume that interest rates are time-varying, but expected excess log returns to all risky assets are constant over time. An investor can invest in two risky assets: equity and long maturity bonds. The log return to equity is denoted by r^{equity}, while the log return to long maturitiy bonds is denoted by r^{bonds}. All log returns are conditionally normal with a constant covariance matrix Ω. The investment opportunity set, as a function of the gross risk-free rate R^f_t is given by:

$$\mu = \begin{bmatrix} E[r^{equity}_{t+1}] + \sigma^2(r^{equity}_{t+1}) \\ E[r^{bonds}_{t+1}] + \sigma^2(r^{bonds}_{t+1}) \end{bmatrix} = \ln R^f_t + \begin{bmatrix} 0.05 \\ 0 \end{bmatrix} \qquad \Omega = \begin{bmatrix} (1/2)^2 & 0 \\ 0 & (1/20)^2 \end{bmatrix}$$

The current risk free rate is given by

$$R^f_t = 1.02.$$

The realized return to the two stocks are uncorrelated with changes in the interest rate, while the return to long term bonds has a negative covariance with changes to the risk-free rate of -0.001.

(a) Assume an investor has a time invariant value function given by:

$$V(W_t, R^f_t) = \frac{1}{1-\gamma}(R^f_t)^{1-\gamma} W_t^{1-\gamma}$$

Find the investors optimal portfolio as a function of γ using the formula[3]

$$w = \left(-\frac{V_w}{V_{ww}W}\right)\Omega^{-1}(\mu - r_f) + \left(-\frac{V_{ws}}{V_{ww}W}\right)\Omega^{-1}\Phi$$

Give your answer in terms of myopic demand and hedging demand.

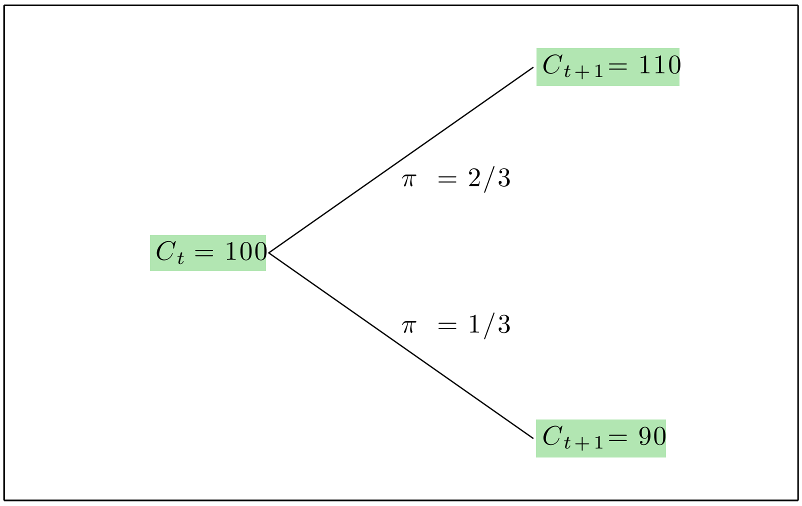
[3] Φ denotes the vector of covariances of risky-asset returns and changes to the state variable R^f, V_w and V_{ww} denote the first and second derivative of the value function with respect to wealth, respectively and V_{ws} denote the cross-derivative of the value function with respect to wealth and the state R^f_t.

(b) Find the hedging demand of three investors, A, B, and C. A has $\gamma = 0.5$, B has $\gamma = 1$, and C has $\gamma = 2$. Explain economically why they differ in the way they do.

9. Consider the following simple economy. The probability of going to the up state next period is equal to 2/3. Assume the representative investor has a utility function given by

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma},$$

with $\gamma = 2$ and he has a time discount factor of $\theta = 0.99$. Investors can trade freely in Arrow-Debreu securities for both states of the world next period.



(a) Assume that the values for C_t and C_{t+1} given in the figure are the equilibrium consumption levels of the representative investor. Find the implied values of:

i. The stochastic discount factor between time t and $t+1$ for both possible states of the economy.

ii. The price of an Arrow-Debreu security that pays out 1 in the upstate and the price of an Arrow-Debreu security that pays out 1 in the downstate.

iii. The price at t of a security that pays out 1 in both possible states of the economy at time $t+1$. (A discount bond.)

(b) Find the risk-neutral probabilities of each state and give an economic interpretation why they differ from the true probabilities in the way they do.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \qquad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - (E[x])^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

$$x_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process

$$x_t = c + \rho x_{t-1} + \epsilon_t \qquad \text{with } |\rho| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

$$E[x_t] = c/(1 - \rho)$$
$$\text{var}\,(x_t) = \frac{1}{1 - \rho^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\rho^j}{1 - \rho^2}\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \rho^j \qquad \text{(auto-correlation)}$$

VR_k statistic

$$VR_k = \frac{1}{k}\frac{\text{var}\,(r_{t,t+k})}{\text{var}\,(r_{t+1})} = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\phi_j,$$

where ϕ_j is the jth autocorrelation coefficient of returns.