Solution Key, Asset Pricing 4.1, October 24, 2011

1. (a)

$$w = \frac{1}{\lambda}\Omega^{-1}\left(E_t[R_{t+1}] - R^f\iota\right) = \frac{5}{6}\begin{bmatrix}1/9 & 1/18\\ 1/18 & 1/9\end{bmatrix}^{-1}\begin{bmatrix}0.06\\ 0.04\end{bmatrix} = \frac{5}{6}\begin{bmatrix}12 & -6\\ -6 & 12\end{bmatrix}\begin{bmatrix}0.06\\ 0.04\end{bmatrix} = \begin{bmatrix}0.4\\ 0.1\end{bmatrix}$$

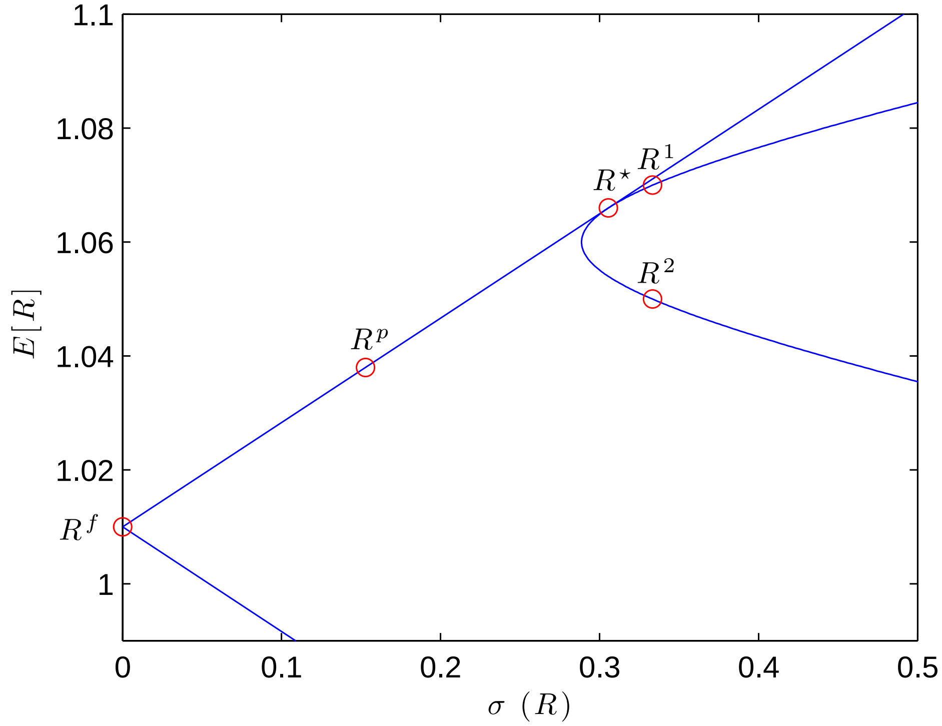
So 40 % to the first stock and 10 % to the second.

(b)

$$E[R^p] = 1.038 \qquad \sigma(R^p) = 0.1528$$

(c) (Double the excess return and the standard deviation of investor portfolio).

$$E[R^\star] = 1.066 \qquad \sigma(R^p) = 0.3055$$



(d)

(e) The market capitalization of the first stock needs to be four times that of the second stock, so that the market and tangency portfolios have the same weights.

2. An arbitrageur with a short investment horizon cannot be sure that prices will move against fundamental and wipe out his position. (Specifically, noise traders might take positions that make prices move prices the wrong way). E.g. anyone going long the relatively undervalued stock and short the overvalued stock at the beginning of the sample would need to be able to hold the position for 3 years before getting in the blue. (*Cheap answers, such as "prices are not equal to fundamentals, so there must be limits to arbitrage" only get partial credit. You need an answer that is specific to short term arbitrageurs, since this is what is asked.*)

3. Sort all stocks based on their book/market ratio in previous year into bins. The factor is the net holding return from buying a portfolio of the stocks from the bin with the highest book/market ratio (value stocks), financed by shorting a portfolio of the stocks from the bin with the lowest book/market ratio (growth stocks). (To get the points here you really need to say precisely what the factor is, i.e. the return to this long-short position.)

4. To get the full points here, you need an answer that:

   (a) gives some reason for why stocks were more popular in the 50s. (E.g. initial high returns, investment clubs, social dynamics, feedback loops)

   (b) link the popularity to flows into the stock market. (Increased participation, the mechanism that translates the increased popularity to higher valuations).

   (c) give some reason why stocks became less popular in the 70s. (E.g. initial losses due to fundamental events.)

   (d) link the loss of popularity with flows out of the stock market. (The mechanism that translates the decreased popularity to lower valuations.

5. (a) i. $E_t[r_{t,t+k}] = k\mu$, so grows linearly with k.

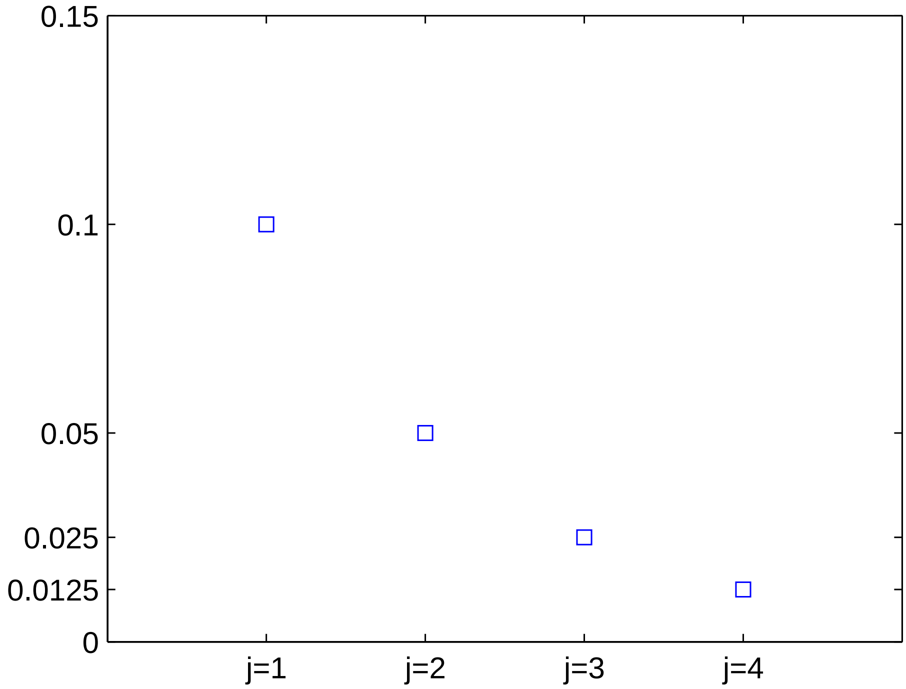
   ii. $\sigma^2(r_{t,t+k}) = k\sigma^2$, so grows linearly with k

   iii. $SR_k = \frac{E_t[r_{t,t+k} - kr^f]}{\sigma(r_{t,t+k})} = \frac{k(\mu - r^f)}{\sqrt{k}\sigma} = \sqrt{k}SR_1$, increases with k.

   (b) This is just what we see from the part (iii) of (a). It does not imply that equity is less risky for the long horizon. That would be the case if the standard deviation grew with *less* than the square root of the holding period.

   (c) At time $t+1$, both investors have wealth of 75 Euros. $w_{t+1} = 1/3$ for investor A and $w_{t+1} = 1/2$ for investor B. Their realized wealth at time $t+2$ is given by:

$$W^A_{t+2} = (2)(25) + 50 = 100$$
$$W^B_{t+2} = (2)(37.5) + 37.5 = 112.5$$



6. (a)
   (b)

$$r_{t+1} - E_t[r_{t+1}] = 0.1 + (0.95)(0.5)(0.1) + \rho^2(0.5)^2(0.1) + \cdots = \frac{1}{1-0.475}0.1 = 0.1905$$

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   (c) All equal to 1, since log stock returns are going to be IID with mean $E[r_t]$ and standard deviation: $0.01/(1-0.475)$

7. (a)

$$E_t[R^i_{t+1} - X^i_{t+1}] = R^f + \left(\frac{\text{cov}(R^i_{t+1}, R^m_{t+1})}{\sigma^2_m} + \frac{\text{cov}(X^i_{t+1}, X^m_{t+1})}{\sigma^2_m} - \frac{\text{cov}(R^i_{t+1}, X^m_{t+1})}{\sigma^2_m)} - \frac{\text{cov}(X^i_{t+1}, R^m_{t+1})}{\sigma^2_m}\right)\lambda_t$$

   (b) i. Investors require a premium to hold stocks that are illiquid at the same time as the market.
   ii. Investors are willing to accept a discount for holding stocks that have high returns when the market is illiquid.
   iii. Investors are willing to accept a discount for holding stocks that are liquid in down-markets.

8. (a) Preliminary calculations:

$$V_W = (R^f_t)^{1-\gamma} W_t^{-\gamma}$$
$$V_{WW} = -\gamma (R^f_t)^{1-\gamma} W_t^{-\gamma-1}$$
$$V_{WS} = (1-\gamma)(R^f_t)^{-\gamma} W_t^{-\gamma}$$

So

$$-\frac{V_W}{V_{WW}W} = -\frac{(R^f_t)^{1-\gamma}W_t^{-\gamma}}{-\gamma(R^f_t)^{1-\gamma}W_t^{-\gamma-1}W_t} = \frac{1}{\gamma}$$
$$-\frac{V_{WS}}{V_{WW}W} = -\frac{(1-\gamma)(R^f_t)^{-\gamma}W_t^{-\gamma}}{-\gamma(R^f_t)^{1-\gamma}W_t^{-\gamma-1}W_t} = \frac{(1-\gamma)}{\gamma R^f_t}$$

Plugging into the equation for optimal portfolio weights we find

$$\begin{aligned}\omega &= \frac{1}{\gamma}\begin{bmatrix}4 & 0\\ 0 & 400\end{bmatrix}\begin{bmatrix}0.05\\ 0\end{bmatrix} + \frac{(1-\gamma)}{\gamma R_t^f}\begin{bmatrix}4 & 0\\ 0 & 400\end{bmatrix}\begin{bmatrix}0\\ -0.001\end{bmatrix}\\ &= \frac{1}{\gamma}\begin{bmatrix}0.2\\ 0\end{bmatrix} + \frac{(1-\gamma)}{\gamma R_t^f}\begin{bmatrix}0\\ -0.4\end{bmatrix}\end{aligned}$$

(b) The hedging demand is -0.392 for investor A, 0 for investor B, and 0.196 for investor C. Investor A has a low level of risk aversion and cares mostly about having more money to invest when expected returns are high. The short position in long maturity bonds has a high return when the risk-free rate increases, which is exactly when expected returns are high. Investor C is sufficiently risk averse that he cares more about the potential downside of having less wealth when investment opportunities are bad. Investor B, as a log utility investor, cares equally about his wealth in states with bad and good investment opportunities. *All these investors are risk-averse! Many of you seem to think that A is risk-seeking and that B is risk-neutral. Risk-neutral would be* $\gamma = 0$*, risk-seeking would be* $\gamma < 0$*. Another common conceptual mistake is to think that the hedging portfolio hedges against losses to the equity position. It does not. It's a pure interest rate hedge.*

9. (a) Assume that the values for C_t and C_{t+1} given in the figure are the equilibrium consumption levels of the representative investor. Find the implied values of:

   i.
   $$m_{t+1} = \theta\left(\frac{C_{t+1}}{C_t}\right)^{-2} = \begin{bmatrix}= 0.8182\\ = 1.2222\end{bmatrix}$$

   ii. $q_1 = (2/3)(0.8182) = 0.5455$, $q_2 = (1/3)(1.2222) = 0.4074$

   iii. $p = q_1 + q_2 = 0.9529$.

   (b) The risk-neutral probability of going up is given by $\tilde{\pi} = \frac{q_1}{q_1+q_2} = 0.5724$. This is lower than the underlying probability of going up. Conversely, the risk-neutral portability of going down is $(1 - 0.5724) = 0.4376$ which is higher than the underlying probability of going down. The risk-neutral probabilities are shifted towards bad states of the world. (I.e. states where marginal-utility and hence also the discount-factor is high).