vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Investments 4.1
Code:	60412040
Coordinator:	Frode Brevik
Date:	December 16, 2010
Time:	15:15
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 part questions (numbered (a), (b), (c))
Type of questions:	Open
Answer in:	English
Credit score:	Each part question give 0.5 of the total grade. Some part questions are divided into subparts numbered with (i), (ii), (iii)
Grades:	Final grades will be made public no later than December 24.
Inspection:	Preliminary set for December 22, at 16:00. Check Blackboard for final time.
Number of pages:	5 (including front page and formula sheet)

1. An investor who is a mean-variance optimizer wants to allocate her portfolio optimally between two stocks and risk free bonds. The returns to the two stocks are jointly normally distributed with $R_{t+1} \sim \mathcal{N}(\mu, \Omega)$. Expected returns are given by

$$R^f = 1.02 \qquad E_t[R_{t+1}] = \begin{bmatrix} 1.07 \\ 1.01 \end{bmatrix}$$

The return to each of the two stocks has a standard deviation of 30 %. The returns to the two stock returns are negatively correlated with a correlation coefficient of -0.3.

$$E[R^p_{t+1}] - \frac{\lambda}{2}\sigma^2(R^p_{t+1})$$

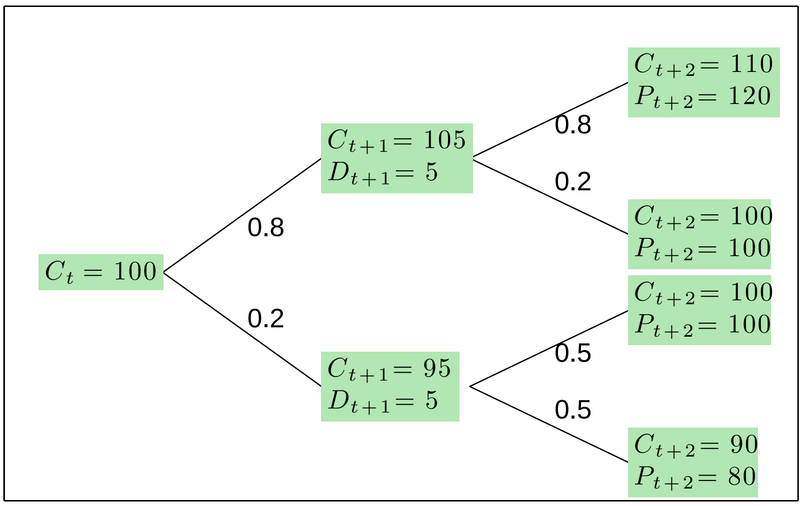
where $E[R^p_{t+1}]$ and $\sigma^2(R^p_{t+1})$ are the expected return and the variance of the return to the investors portfolio.

   (a) What percentage of her wealth should an investor with $\lambda = 2$ allocate to each of the two stocks and what percentage to risk-free bonds.
   (b) Asset 2 is risky and has a lower expected return than risk-free bonds. Why can it still make sense for the investor to hold it?
   (c) Compute the expected return and standard deviation of the investor's portfolio.
   (d) Compute the expected return and standard deviation the tangency portfolio between the risky-asset frontier and the mean-variance frontier.
   (e) Sketch the Risky-asset frontier and the mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:
      i. The Investor's portfolio.
      ii. The tangency portfolio.
      iii. Risk-free bonds.
      iv. The two stocks.
   (f) Assume that the market portfolio has 50% allocated to each of the two stocks. Compute the expected return and variance of the market portfolio, the betas of the two stocks with the market portfolio, and their expected return according to the CAPM.

2. Consider the following simple economy. The probabilities of going up or down depend on the state of the economy. In the upstate at time $t+1$, the probabilities are 0.8 and 0.2, the same as at time t, while in the downstate, both probabilities are 0.5. The representative investor has a utility function given by

$$U(C) = \ln C,$$

and he has a time discount factor of $\theta = 0.95$.



(a) Assume that the values for C_t and C_{t+1} given in the figure are the equilibrium consumption levels of the representative investor. Find the implied values of the stochastic discount factor between time t and $t+1$, and times $t+1$ and $t+2$ for all possible states of the economy.

(b) The numbers given for P_{t+2} are the prices of a particular stock in different states of the economy at time $t+2$. This stock pays a dividend of 5 at time $t+1$. Find the ex-dividend price of the stock at time in both states at time $t+1$ and and at time t.

(c) Argue whether the expected return to holding the stock should be higher or lower than the risk-free rate based on the covariance of the stock return with consumption. (Argue economically. You don't need to calculate expected returns or risk-free rates.)

3. Campbell and Shiller decompose unexpected stock returns into

$$r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}$$

Assume $\rho = 0.95$. Let log dividend growth be given by

$$\Delta d_{t+1} = \epsilon_{t+1} - 0.5\epsilon_t, \qquad \epsilon \sim I.I.D, \mathcal{N}(0, (0.1)^2).$$

Moreover, let the required return to equity be given by:

$$E_t[r_{t+1}] = r + x_t$$

where

$$x_{t+1} = 0.9x_t + \nu_{t+1} \qquad \nu_{t+1} \sim I.I.D, \mathcal{N}(0, (0.05)^2)$$

(a) Assume $\epsilon_{t+1} = -0.2$. Sketch how expected log dividend growth rates (Δd_{t+j}) change from time t to $t+1$ for $j = \{1, 2, 3, 4\}$

(b) Assume $\nu_{t+1} = 0.1$. Sketch how expected stock returns (r_{t+1+j}) change from t to $t+1$ for $j = \{1, 2, 3, 4\}$

(c) Compute the contribution of (i) dividend growth news (cash flow news) and (ii) news about future expected returns (discount rate news) to the unexpected return at time $t+1$.

4. Let the log return to equity be given by the time invariant equation

$$r_{t+1} = 0.1 + \epsilon_{t+1}, \qquad \epsilon_{t+1} = \mathcal{N}(0, 0.2^2)$$

and ϵ_{t+1} is I.I.D.

(a) Compute the variance ratios VR_1 and VR_2.

(b) Assume a power utility investor wants to divide her wealth between equity and risk-free bonds earning a constant return $r^f = 0.05$. How should the investor change her allocation to equity as she gets older given the process for equity returns above?

(c) Assume she optimally allocated 50 % of her wealth to equity at time t. At time $t+1$, $\epsilon_{t+1} = -0.4$. Give the portfolio weights of the investor at time $t+1$ before she has rebalanced her portfolio.

(d) Describe what trades she needs to undertake at time $t+1$ to optimally rebalance her portfolio.

5. Assume that the representative investor is a log utility maximizer with period utility function $U(C) = \log C$ and time discount factor is $\theta = 0.99$. The economy can be in either of two states, a boom (state 1) or a recession (state 2). Quarterly consumption growth is conditionally log-normal, with

$$\Delta c_{t+1} = \mu_{t+1} + \sigma\epsilon_{t+1}, \qquad \epsilon_{t+1} \sim \mathcal{N}(0, 1)$$

$$\mu_{t+1} = \begin{cases} 0.005, & \text{if } s_{t+1} = 1 \\ -0.005, & \text{if } s_{t+1} = 2 \end{cases}$$

$$\sigma = 0.005$$

The probability of going from a boom to a recession between two quarters is 0.05, that of going from a recession to a boom is 0.25.

(a) Use the law of iterated expectations to find an expression for $E_t[m_{t+1}|S_t = 1]$ in terms of the transition probabilities from the boom state and the expected value of the discount factor conditional on the two possible states at time $t + 1$.

(b) Compute

$$\bar{m}_1 = E[m_{t+1}|s_{t+1} = 1] \text{ and}$$
$$\bar{m}_2 = E[m_{t+1}|s_{t+1} = 2],$$

the expectation of the stochastic discount factor between time t and $t + 1$, conditional on ending in state 1 or 2.

(c) Compute $R^{(1)}$, the one period interest rate in state 1 and 2.

(d) Explain in economic terms why the interest rate is higher in one of the states than in the other.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & 0 & \dots & 0 \\ 0 & a_{2,2} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & a_{n,n} \end{bmatrix}^{-1} = \begin{bmatrix} 1/a_{1,1} & 0 & \dots & 0 \\ 0 & 1/a_{2,2} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & 1/a_{n,n} \end{bmatrix} \quad \text{(Diagonal matrix)}$$

Expectations, variances

$$E_t[ax + by] = aE_t[x] + bE_t[y] \quad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \quad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - E[x]^2$$
$$\text{var}\,(ax) = a^2 \text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

If

$$x_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \dots + \theta_q\epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

Then

$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \dots + \theta_q^2)\sigma^2 \quad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \dots + \theta_{q-j}\theta_q)\sigma^2 \quad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \dots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \dots + \theta_q^2} \quad \text{(auto-correlation)}$$

AR(1) process

If

$$x_t = \mu + \rho x_{t-1} + \epsilon_t \qquad \text{with } |\rho| < 1, \epsilon_t \sim \mathcal{N}(0, \sigma^2),\ \epsilon_t\ I.I.D.$$

Then

$$\text{var}\,(x_t) = \frac{1}{1 - \rho^2}\sigma^2 \quad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\rho^j}{1 - \rho^2}\sigma^2 \quad \text{(auto-covariance)}$$
$$\phi_j = \rho^j \quad \text{(auto-correlation)}$$

VR_k statistic

$$VR_k = \frac{1}{k}\frac{\text{var}\,(r_{t,t+k})}{\text{var}\,(r_{t+1})} = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\phi_j,$$

where ϕ_j is the jth autocorrelation coefficient of returns.