Investments 4.1
Course code: 60412040

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Date: December 10, 2009
Time: 15:15 – 18:00
(2 hours, 45 minutes)

Exam form:	This is a closed book exam. Calculators including graphical calculators are allowed.
Parts:	The exam contains 20 parts. Some parts are divided into subparts. The subparts are numbered by (i), (ii), etc.
Grading:	Each part count equally to the exam grade.
Results:	Results will be posted on Blackboard at the latest by Thursday, December 17.
Inspection:	You can inspect your marked exam papers Thursday, December 17, 16:15. The room will be announced.
Remark:	Provide complete answers, including computations where appropriate. On verbal questions: always provide motivation/explanation of your answer in terms of the economic mechanism at play. A short "yes" or "no" will never do as an answer. But be concise in your answers, otherwise you'll loose too much time writing them down.

Notice formula sheet at the end!

1. An investor who is a mean-variance optimizer wants to allocate her portfolio optimally between two stocks and risk free bonds. The returns to the two stocks are jointly normally distributed with $R_{t+1} \sim \mathcal{N}(\mu, \Omega)$. Expected returns are given by

$$R^f = 1.02 \qquad E_t[R_{t+1}] = \begin{bmatrix} 1.12 \\ 1.06 \end{bmatrix}$$

The return of each risky asset has a standard deviation of 50 % and the returns to the two assets are uncorrelated. The investor chooses a vector of portfolio weights w for the two stocks to maximize:

$$w' E_t[R_{t+1}] + (1 - w'\iota) R^f - \frac{\lambda}{2} w' \Omega w$$

(a) The optimal portfolio weights w for the risky assets are given by

$$\frac{1}{\lambda} \Omega^{-1} (E_t[R_{t+1}] - R^f \iota)$$

i. What percentage should an investor with $\lambda = 1$ allocate to risk free bonds?

ii. How does the allocation to risk-free bonds change as λ increases. (Argue why in terms of the investors preferences.)

(b) Compute the expected return and standard deviation of the investor's portfolio.

(c) Compute the mean and standard deviation of the tangency portfolio between the risky-asset frontier and the mean-variance frontier.

(d) Sketch the Risky-asset frontier and the mean-variance frontier in the standard standard-deviation/expected returns diagram. Make sure to label the axis and mark the points corresponding to:

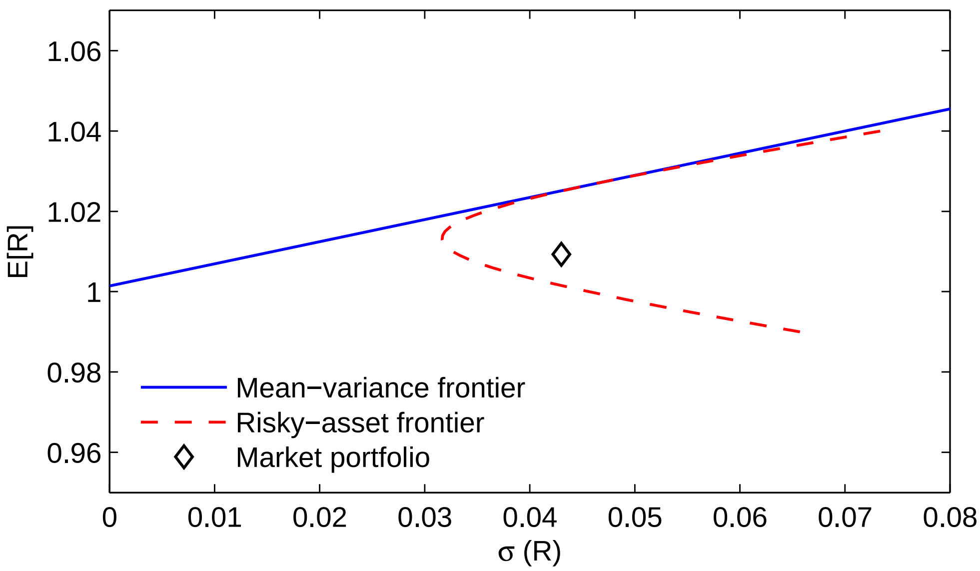
i. The Investor's portfolio.

ii. The tangency portfolio.

iii. Risk-free bonds.

iv. The two stocks.

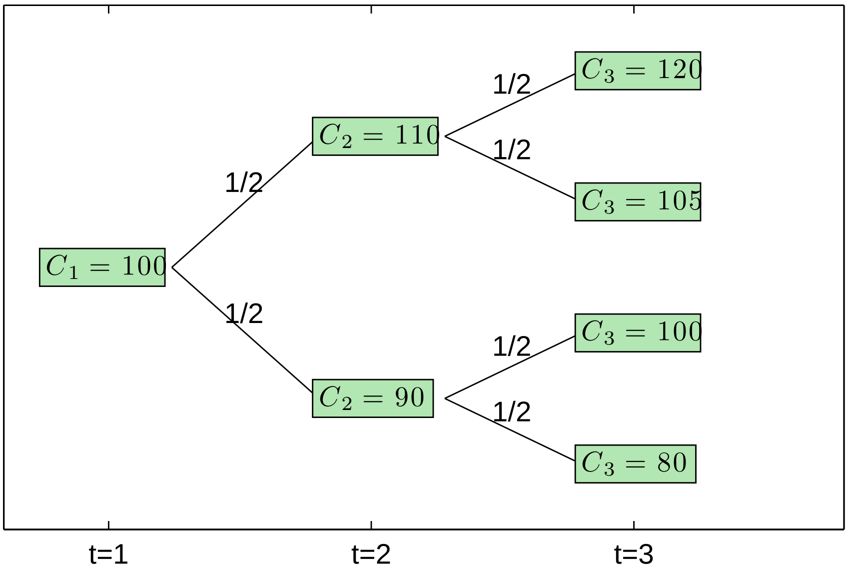
2. This figure is taken from your first assignment. The risky-asset frontier is computed by assuming that the expected return to each of the risky assets available are equal to their historical averages. The point marked in the figure gives the expected return and the standard deviation of the market portfolio.



(a) Is the situation given in the figure compatible with the CAPM? (Could the CAPM hold here?) Justify your answer.

(b) Copy the figure to your answer sheet and sketch how the figure would change in equilibrium if all investors were mean-variance optimizers. Assume that both the risk-free rate and the expected return and standard deviation of the market portfolio stay the same.

3. Consider the following simple economy. The probability of going up is equal to 1/2 always. C_t is consumption in year t depending on the state of the economy.



(a) Assume the representative investor has a period utility function given by

$$U(C) = \ln C$$

and a time discount factor of $\theta = 1$. Find the discount factor m_{t+1} at each node in the two years $t = 2$ and $t = 3$.

(b) Explain economically why the discount factor is higher in the "down" state then in the "up" state at $t = 2$.

(c) Use your results from (a) to find

i. the price at $t = 1$ of risk-free one period bond that pays out 1 at time $t = 2$.

ii. the prices in both possible states at time $t = 2$ of a bond that pays out 1 at time $t = 3$.

iii. The price at $t = 1$ of a zero-coupon bond that pays out 1 at $t = 3$.

(d) Find the one and two period risk-free rates at $t = 1$.

4. The log return to a particular stock is given by

$$r_{t+1} = 0.05 + \epsilon_{t+1} - 0.1\epsilon_t \qquad \epsilon_{t+1} = \mathcal{N}(0, 0.2^2) \tag{1}$$

and ϵ_t I.I.D.

(a) Compute

$$\frac{E[r_{t+1}]}{\text{var}(r_{t+1})} \quad \text{and} \quad \frac{E[r_{t,t+2}]}{\text{var}(r_{t,t+2})}$$

As well as the VR_1 and VR_2 statistics.

(b) How would you expect the average static allocation to equity differ between an investor with a 1 year investment horizon and an investor with a 2 year investment horizon based on your result above. Provide an economic explanation. (Assume both investors have a power utility function.)

(c) Assume $\epsilon_{t-1} = 0$, $\epsilon_t = 0$ and $\epsilon_{t+1} = 0.2$. Should a power utility investor with a 1 year investment horizon allocate more or less to equity at time $t+1$ than she did at time t? Provide an argument in terms of the risk-return trade-off the investor faces at t and $t+1$.

5. The Campbell-Shiller decomposition of expected stock returns is given by

$$r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}$$

Let log dividend growth and expected returns be given by:

$$\Delta d_t = 0.01 + \epsilon_t - 0.5\epsilon_{t-1} \qquad \epsilon_t = \mathcal{N}(0, 0.01^2)$$
$$E_t[r_{t+1}] = 0.01 + 0.9E_{t-1}[r_t] + \nu_t \qquad \nu_t = \mathcal{N}(0, \sigma_2^2)$$

and ν_t and ϵ_t I.I.D.

(a) i. Identify the two terms "discount rate news" and "cash flow news" in the Campbell-Shiller decomposition above. (What part of the right hand side gives "discount rate news"? What part gives "cash-flow news"?)

ii. Give an economic interpretation of each of the two terms.

(b) Give a simplified expression for $\eta_{d,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}$ in terms of ϵ_{t+1}.

(c) Give a simplified expression for $\eta_{r,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta r_{t+1+j}$ in terms of ν_{t+1}.

(d) Assume $\rho = 0.95$.

i. Find the impact of $\epsilon_{t+1} = 0.01$ on unexpected stock returns at time $t+1$.

ii. Find the impact of $\nu_{t+1} = 0.01$ on unexpected stock returns at $t+1$.

6. Assume that excess log return to equity is given by:

$$r^e_{t+1} = r_{t+1} - r^f = 0.03 + 2\ dp_t + \epsilon_{t+1}, \qquad \epsilon_{t+1} = \mathcal{N}(0, 0.2^2),$$

where dp_t is the log dividend yield at time t. The conditional covariance of the dividend yield with excess equity returns is given by

$$\text{cov}_t\left(r^e_{t+1}, dp_{t+1}\right) = -0.001$$

An investor's value function at time t is given by

$$V(t, W, dp) = (dp_t)^{1-\gamma} \frac{W_t^{1-\gamma}}{1-\gamma}$$

(a) Give a possible economic reason why the value function of the investor should depend on the current log dividend yield. (Why should the investor be better off when the dividend yield is high?)

We saw in class, that the optimal total equity allocation of the investor is given by:

$$w = \left(-\frac{V_W}{V_{WW}W}\right) \frac{E_t[r^e_{t+1}]}{\text{var}_t\left(r^e\right)} + \left(-\frac{V_W}{V_{WW}W}\right) \frac{V_{W,dp}\ \text{cov}_t\left(r^e, dp_{t+1}\right)}{V_W\ \text{var}_t\left(r^e\right)}$$

Assume $dp_t = 0.1$.

(b) Compute the myopic demand of the investor for $\gamma = 4$.

(c) Compute the hedging demand of the investor for $\gamma = 4$.

Important formulas

Vector derivatives

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x$$
$$\frac{\partial x'Ax}{\partial x} = (A + A')x$$

Inverses

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/d \end{bmatrix} \qquad \text{(Special case)}$$

Expectations, variances

$$E_t[a + bx] = a + bE[x] \qquad \text{(Linearity)}$$
$$E_t[E_{t+1}[x]] = E_t[x] \qquad \text{(L.I.E.)}$$
$$\text{var}\,(x) = E[x^2] - E[x]^2$$
$$\text{var}\,(ax) = a^2\text{var}\,(x)$$
$$\text{cov}\,(x, y) = E[x\ y] - E[x]E[y]$$
$$\text{cov}\,(ax, y) = a\text{cov}\,(x, y) = \text{cov}\,(x, ay)$$

Expectation of a log-normal

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \qquad \text{if} \qquad x \sim \mathcal{N}(\mu, \sigma^2)$$

MA(q) processes

If

$$x_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \cdots + \theta_q\epsilon_{t-q} \qquad \text{with } \epsilon_t \text{ I.I.D.}$$

Then

$$\text{var}\,(x_t) = (1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2)\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = (\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q)\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \frac{\theta_j + \theta_1\theta_{j+1} + \theta_2\theta_{j+2} + \cdots + \theta_{q-j}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2} \qquad \text{(auto-correlation)}$$

AR(1) process

If

$$x_t = \mu + \rho x_{t-1} + \epsilon_t \qquad \text{with } \epsilon_t \sim \mathcal{N}(0, \sigma),\ \epsilon_t\ I.I.D.$$

Then

$$\text{var}\,(x_t) = \frac{1}{1 - \rho^2}\sigma^2 \qquad \text{(variance)}$$
$$\text{cov}\,(x_t, x_{t-j}) = \frac{\rho^j}{1 - \rho^2}\sigma^2 \qquad \text{(auto-covariance)}$$
$$\phi_j = \rho^j \qquad \text{(auto-correlation)}$$

VR_k statistic

$$VR_k = \frac{1}{k}\frac{\text{var}\,(r_{t,t+k})}{\text{var}\,(r_{t+1})} = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\phi_j,$$

where ϕ_j is the jth autocorrelation coefficient of returns.