Investments 4.1

Solution key, retake exam

December 10, 2009

1. (a) i. 0.44
      ii. It increases, as λ increases, the investor becomes more risk averse and wants to keep more of his portfolio in risk-free bonds.
   (b) 1.0664, 0.2154
   (c) 1.1029, 0.3847
   (d) i. At MVF, 60 % of the way between $(0, R^f)$ and the tangency portfolio.
      ii. At tangency point.
      iii. At $(0, R^f)$
      iv. First is on RAF above the tangency point, second is on RAF below the tangency point.

2. (a) No, for the CAPM to hold, we need the market portfolio to be at the tangency point.
   (b) The slope of the MVF decreases enough so that the MVF goes through the point of the market portfolio. The point of the market portfolio is now also the tangency point between the RAF and the MVF.

3. (a) From top to bottom:

$$m_2 = \begin{bmatrix} 1/1.1 \\ 1/0.9 \end{bmatrix} \qquad m_3 = \begin{bmatrix} 11/12 \\ 11/10.5 \\ 9/10 \\ 9/8 \end{bmatrix}$$

   (b) Consumption is lower in the down state, so one unit of extra consumption is more valuable here.
   (c) i. 1.0101;
      ii. 0.9821 and 1.0125;
      iii. 1.0089 (Calculate by using price of 1 period bond at $t+1$, that is by $(0.5)(1/1.1)(0.9821)+(0.5)(1/0.9)(1.0125)$, or take expectation of product $(m_2)(m_4)$.)
   (d) 0.99 and 0.9955 $(= 1/\sqrt{1.0089})$.

4. (a)

$$\frac{E[r_{t+1}]}{\text{var}\,(r_{t+1})} = \frac{0.05}{(1.01)(0.2)^2} = 1.2376 \qquad \frac{E[r_{t,t+2}]}{\text{var}\,(r_{t,t+2})} = \frac{0.1}{(1.82)(0.2)^2} = 1.3736$$

$$VR_1 = 1 \qquad VR_2 = 1 + (-0.1)/(1+0.1^2) \approx 0.9$$

   (b) The 2 year investor should invest more, because she faces a more attractive risk-return trade-off.
   (c) Expected returns are higher at $t+1$, while the conditional variance is the same, so she should invest more in equity.

5. (a) i. The first sum gives "cash-flow news". The second sum gives "discount rate news".
      ii. "Cash-flow news" gives the part of unexpected stock returns that are due to changes in expectations about future cash-flows on the stock. "Discount rate news" gives the part of unexpected returns that are due to changes in required rates of returns on the stock.

(b) $\eta_{d,t+1} = \rho^0 \epsilon_{t+1} - \rho^1(0.5)\epsilon_{t+1} = (1 - 0.5\rho)\epsilon_{t+1}$

(c) $\eta_{r,t+1} = \rho^1 \nu_{t+1} + \rho^2 0.9 \nu_{t+1} + \rho^3 (0.9)^2 \nu_{t+1} + \ldots = \frac{\rho}{1-0.9\rho} \nu_{t+1}$.

(d) i. $(1 - (0.95)(0.5))(0.01) = 0.00525$.

ii. $(0.95)/(1 - (0.95)(0.9))(0.01) = 0.0655$.

6. (a) When the dividend yield is high, expected returns are also high, so the expected portfolio return of the investor is also higher. This increases expected future consumption and leaves the investor better off.

(b) $\left(-\frac{V_W}{V_{WW}W}\right) \frac{E_t[r^e_{t+1}]}{\text{var}_t(r^e)} = \frac{1}{4}\frac{0.23}{0.04} = 1.4375$.

(c) $\left(-\frac{V_W}{V_{WW}W}\right) \frac{V_{W,dp}}{V_W} \frac{\text{cov}_t(r^e, dp_{t+1})}{\text{var}_t(r^e)} = \frac{1}{4}\frac{(1-4)(-0.001)}{(0.1)(0.04)} = 0.1875$.