Investments 4.1
Course code: 60412040

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Date: October 22, 2008
Time: 15:15 – 18:00
(2 hours, 45 minutes)

Parts:	The exam contains 20 parts. Some parts are divided into subparts. The subparts are numbered by (i), (ii), etc.
Grading:	Each part will give 3.5 points of the total 70.
Results:	Results will be known as soon as possible. At the latest Tuesday November 4.
Inspection:	You can inspect your marked exam papers Thursday, November 6, 15:30. The room will be announced.
Remark:	Provide complete answers, including computations where appropriate. On verbal questions: always provide motivation/explanation of your answer in terms of the economic mechanism at play. A short "yes" or "no" will never do as an answer. But be concise in your answers, otherwise you'll loose to much time writing it down.

Scan for the easiest questions and solve them first!

Useful formulas

$$\frac{\partial x'a}{\partial a} = \frac{\partial a'x}{\partial a} = x \tag{1}$$

$$\frac{\partial x'Ax}{\partial x} = (A + A')x \tag{2}$$

$$\text{Var}\,(x) = E[x^2] - E[x]^2 \tag{3}$$

$$\text{Cov}\,(x, y) = E[x \cdot y] - E[x]E[y] \tag{4}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{5}$$

If $x \sim \mathcal{N}(\mu, \sigma^2)$, then

$$E[e^x] = e^{\mu + \frac{1}{2}\sigma^2} \tag{6}$$

1. An investor who is a mean-variance optimizer wants to allocate her portfolio optimally between to stocks and risk free bonds. The returns to the two stocks are jointly normally distributed with $R_{t+1} \sim \mathcal{N}(\mu, \Omega)$. The investment opportunity set faced by investor is characterized by

$$R_f = 0.05 \qquad \mu = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0.04 & -0.01 \\ -0.01 & 0.04 \end{bmatrix}$$

The investor chooses a vector of portfolio weights w for the two stocks to maximize:

$$w'\mu + (1 - w'\iota)R_f - \frac{\lambda}{2} w'\Omega w$$

(a) Compute the optimal w for an investor with $\lambda = 2$.

(b) Compute the expected return and variance of the investor's portfolio.

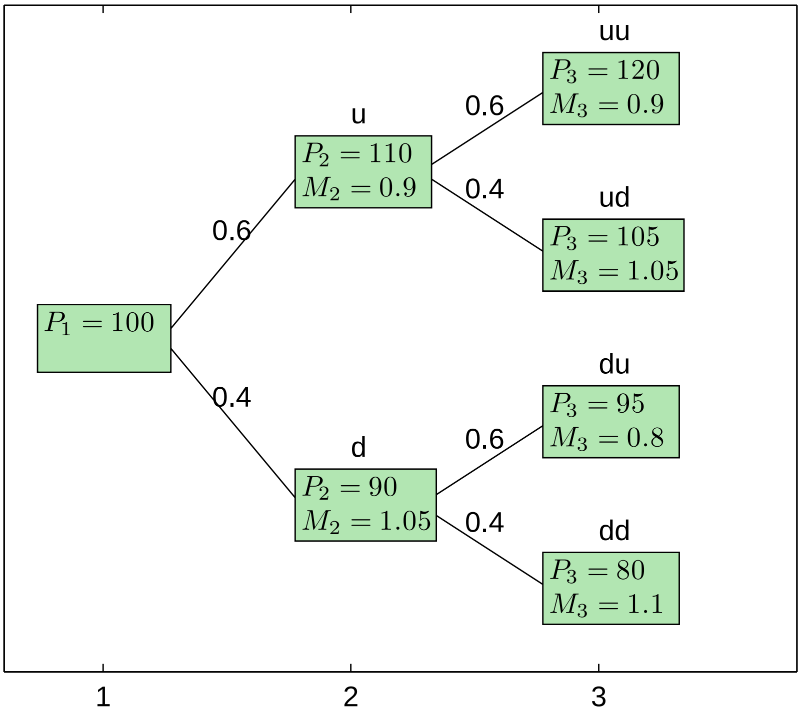
(c) The second stock has an expected return which is equal to the risk-free rate. Explain why it can make sense to hold such a stock in your portfolio.

(d) Assume that the CAPM is true and that the value weighted supply of the two stocks are equal. (Value weighted supply = (Number of outstanding shares)×(current price per share)).

i. How would prices have to move in order for market equilibrium to obtain.

ii. How would this affect expected returns on the two stocks?

2. Consider the following simple economy. The probability of going up is equal to 60% always. P_t is the price of a stock in period t and M_{t+1} is the value of the stochastic discount factor between times t and $t+1$ depending on the state of the economy.



(a) Find the interest rate at time 1 and the interest rates in each of the nodes at time 2.

(b) ST_t is the value at time t of a security that at time $t = 3$ pays out $|P_3 - 100|$.[1] Compute the value of the security at the nodes at $t = 2$ and $t = 1$.

3. Let M_{t+1} denote the stochastic discount factor between time t and $t+1$. The Fundamental Asset Pricing Equation states that the return on any asset should be related to the stochastic discount factor M_{t+1}

$$1 = E_t[M_{t+1}(1 + R_{t+1})]$$

i. Use this equation to show that

$$E_t[R_{t+1}] = R_f - \frac{\text{Cov}\,(M_{t+1}, R_{t+1})}{E_t[M_{t+1}]},$$

where R_f is the risk free interest rate.

ii. Explain why expected returns depend negatively on the covariance between returns and the stochastic discount factor.

[1] It would e.g. pay out 0 if $P_3 = 100$ and 10 if $P_3 = 90$ or $P_3 = 110$. Such a payout structure is called a straddle.

4. Let log consumption growth, Δc_{t+1} be given by $\Delta c_{t+1} \sim \mathcal{N}(0.02, 0.01)$ and the stochastic discount factor be given by $M_{t+1} = 0.99e^{-\gamma \Delta c_{t+1}}$, where γ is the coefficient of relative risk aversion.

   (a) i. Find an expression for the continuously compounded risk-free rate (the log interest rate) using the relations $1 + R_f = 1/E_t[M_{t+1}]$ and $e^{r_f} = 1 + R_f$.

      ii. Compute the continuously compounded interest rate for $\gamma = 1$, $\gamma = 5$, and $\gamma = 25$.

   (b) The C-CAPM implies that the excess return of equity over risk free bonds by given by $\gamma \text{Cov}(\Delta c_{t+i}, r_e)$, where r_e is the log return to equity. Assume the correlation coefficient between log equity returns and log consumption growth is 0.1 and that the standard deviation of equity returns is 0.2. Find the expected excess return for $\gamma = 1$, $\gamma = 5$ and $\gamma = 25$.

   (c) Relate your findings above to the equity premium puzzle and the risk-free rate puzzle.

5. In your case 3, we assumed that stock returns and the dividend yield were related through the VAR:

$$\begin{bmatrix} r_{t+1} \\ (D/P)_{t+1} \end{bmatrix} = \begin{bmatrix} -0.05 \\ 0.0033 \end{bmatrix} + \begin{bmatrix} 0.047361 & 3.5058 \\ 0.002202 & 0.875229 \end{bmatrix} \begin{bmatrix} r_t \\ (D/P)_t \end{bmatrix} + \epsilon_{t+1}$$

   Assume the VAR describes the true relation between stock returns and the dividend yield. If your equilibrium model for equity would implies that returns should be IID (identically and independently distributed). What would this relationship between returns and the dividend yield tell you about market efficiency.

6. The variance ratio statistic for horizon k, VR_k, is computed as

$$\text{VR}_k = \frac{1}{k} \frac{\text{Var}(r_{t,t+k})}{\text{Var}(r_{t+1})},$$

   where $r_{t,t+k}$ is the log return to the asset between time t and k (i.e. $r_{t,t+k} = \sum_{j=1}^{k} r_{t+j}$). Let returns be generated by

$$r_{t+1} = 0.05 + \epsilon_t - 0.05\epsilon_{t-1}, \qquad \epsilon_t \sim \mathcal{N}(0, 0.2^2)$$

   (a) Compute the variance ratio statistics VR_2 and VR_3.

   (b) Empirical researchers tend to find that the Variance ratio statistic is higher than one for horizons shorter than 1 year, but lower than one for horizons larger than 1 year.

      i. What does this finding tell us about the riskiness of equity in the long run relative to the short run.

      ii. How should an investor with CRRA utility (power utility), change her allocation to equity as she grows older if she trusts the empirical evidence?

      iii. Compare your recommendation to that of Samuelson.

   (c) Describe a bootstrapping procedure that would let you compute significance levels for VR_k under the null that returns are independent, but with an unknown distribution.

7. The Campbell-Shiller decomposition of unexpected returns is given by:

$$r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j}$$

   (a) What fraction of return variance would be explained by dividend growth rate news if:

      i. Dividends follow a random walk with a drift.

      ii. Returns are I.I.D. (identically and independently distributed).

   (b) Assume there is an unexpected structural break at time T in the required return to equity. Before time T, expected returns are given by $E_t[r_{t+j}] = 0.08$, for all $j > 1$. at T the expected return to equity decreases to 0.02. Investors expect the structural break to be permanent, so that $E_T[r_{T+j}] = 0.02$ for all $j > 1$. Expected dividend growth is unaffected by the structural break. Compute the unexpected return to equity at time T: $(r_T - E_{T-1}[r_T])$ given a coefficient $\rho = 0.99$.

   (c) Assume that at time t the price of equity drops by 50 % but there has been no news about future dividend growth. How does this influence your expectation about future returns according to the Campbell-Shiller decomposition? Can you tell how expected returns at different horizons are affected?

8. An investor's value function at time t is given by

$$J_t(W_t) = \max_{C_t,\omega_t} \log C_t + \theta E_t[J_{t+1}(W_{t+1})]$$

   The time $t+1$ value function is given by

$$J_{t+1}(W_{t+1}) = \frac{1}{1-\theta}\log W_{t+1} + k,$$

   where k is a constant. The investor earns the stochastic return R_{t+1} on the fraction ω_t he invests in stocks and the risk-free rate R_f on what he invests in bonds. His wealth evolves according to $W_{t+1} = (W_t - C_t)(1 + (1-\omega_t)R_f + \omega_t R_{t+1})$.

   (a) Find the optimal time t consumption level as a function of wealth.

   (b) Show that the optimal allocation to stocks $\omega_t^\star$, is independent of the wealth level.

   (c) Substitute for the optimal consumption and $\omega_t^\star$ in the time t value function. Take $\omega_t^\star$ as a given in this exercise. Solve for $J_t(W_t)$ as a function of W_t. (Feel free to group all terms that are independent of wealth into a single variable).