Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration

Programs:	M.Sc. Finance, M.Sc. Quantitative Finance
Exam:	Investments 4.1
Course code:	60412040
Date:	Dec 12, 2007
Time:	12:00 – 14:45
Duration:	2 hours, 45 minutes
Parts:	The exam has 4 questions, 18 subquestions.
Grading:	Each of 18 subquestions in the exam will be equally weighted. Perhaps redundantly: the written exam makes up 70% of your final grade. The remaining 30% is scored by the cases. The exam can be retaken. The cases cannot.
Results:	Results will be made known as soon as possible, but at the latest Wednesday, Jan 2, 2008.
Inspection:	You can inspect your marked exam papers Friday, January 11, 9:00am. The room will be announced via the monitor system.
Remark:	**Be brief, but complete!** Explanation: Provide complete answers (including computations where appropriate). Always provide motivation/explanation of your answer, even if this is not mentioned explicitly in the question. A short ‘yes’ or ‘no’ will never do as an answer. But also be concise/crisp in your answer, or it will take you too much time to write it down. Use your time efficiently.

Scan for the (in your opinion) easier questions first. Good luck!

This document has 6 pages (this page included)

If X~N(m,s^2), then E[exp(X)] = E[e^X] = exp(m + 0.5s^2).

If $X \sim \mathrm{N}(\mu, V)$, then $w'X \sim \mathrm{N}(w'\mu, w'Vw)$.

cov(X,Y) = E[XY] - E[X]E[Y],
var(X) = E[X^2] - E[X]^2,

cov(X,Y) = cov(X, E[Y|X])

$X = \exp(\ln(X))$; $\exp(X)\exp(Y) = \exp(X+Y)$; $\ln(X^a) = a\ln(X)$

$$\frac{\partial \exp(x)}{\partial x} = \exp(x); \quad \frac{\partial x^a}{\partial x} = a \cdot x^{a-1}; \quad \frac{\partial a^x}{\partial x} = a^x \cdot \ln(a);$$

$$\frac{\partial \ln(x)}{\partial x} = x^{-1}; \quad \frac{\partial (c \cdot x + b)^a}{\partial x} = c \cdot a \cdot (c \cdot x + b)^{a-1}$$

Q1:
In a world where investors all have mean variance preferences, work out the allocation – equilibrium – pricing – performance cycle. In particular,

a. write down the objective function of a mean-variance optimizer in a world with **two** risky assets, and 1 riskfree asset, no short-sale restrictions.
   Also state the first order conditions *and* derive from these the optimal asset allocation.
b. Show from your derivation that two-fund separation holds and how.
   What does this imply for the real-life fact that there are many different mutual funds available on the market?
c. Show that in equilibrium, the CAPM pricing equations hold.
d. Describe briefly the Jensen's alpha, Sharpe ratio, and Treynor ratio and their differences.

see book and transparencies

Q2:

a. Explain how to compute the Carhart momentum factor for a sample of 350 European stocks.

sort stocks cross-sectionally over performance over last 6months (or 1Yr), create deciles, for each decile construct the return over the NEXT month, subtract the NEXT month return from the worst past performers from that of the best past performers. Repeat this for every cross-section.

b. You have 3 products that have the following properties in a Fama-French-Carhart style regression (regressions based on monthly data).

	Constant term	β for excess market return	β for size factor	β for value factor	β for momentum factor	St.dev. of regression residuals
Product 1	0.00	1.00	0.00	0.00	0.00	0.1%
Product 2	-0.01	-0.30	0.00	0.00	1.10	1.5%
Product 3	0.01	0.10	0.00	0.00	-0.90	0.8%

The covariance matrix of the four factors in the regression is

	Market	Size	value	momentum
Market	0.29%	0.06%	0.04%	0.12%
Size	0.06%	0.11%	0.01%	0.02%
Value	0.04%	0.01%	0.13%	0.02%
momentum	0.12%	0.02%	0.02%	0.21%

The means of the factors are

Market	size	value	momentum
0.65%	0.19%	0.49%	0.55%

What is the systematic risk of product 2? .09*0.29%+1.21*0.02%-0.66*0.12%
What is its idiosyncratic risk? $1.5\%^2$
What is the interpretation of the constant term in the regression? skill or alpha; return not explained by exposure to systematic risk.
What is the expected return on product 2? -1%-0.3*0.65%+1.1*0.55%

c. Build a portfolio of these three products that has no systematic risk, but **positive** alpha.
What does the portfolio look like? 7.4% in 1, 41.7% in 2, 50.9% in 3.
What is the expected return on this portfolio? 0.509*1% - 0.417*1%
What is its systematic risk? 0%
What is the total risk of the portfolio? $0.074^2 \cdot 0.001^2 + 0.417^2 \cdot 0.015^2 + 0.509^2 \cdot 0.008^2$

d. Describe a bootstrap procedure to obtain a confidence interval for the Sharpe-ratio of your portfolio from question c.

using the above weights, compute the past portfolio returns.
resample from these returns with replacement a new time series of the same length.
compute the SR for the resampled series and store it.
repeat this many times.
look at the 2.5 and 97.5 percentile of the resampled SRs.

Q3:

a. For an agent that maximizes the sum of expected utilities of consumption now and consumption at t=1, derive the first order conditions.
Relate the previous answer to the fundamental asset pricing equation (FAPE), and state the FAPE in its form of expected excess returns of individual assets.

$$\max U(C_0) + E[U(C_1)]$$
$$C_1 = (W_0 - C_0)\alpha'(1+R)$$
$$\iota'\alpha = 1$$

$$U'(C_0) - E[U'(C_1)\alpha'(1+R)] = 0$$
$$E[U'(C_1)(W_0 - C_0)(1+R)] = \lambda\iota$$
$$\iota'\alpha = 1$$

multiply the 2nd by α' and use the 1st and 3rd to obtain (following class notes)

$$E[U'(C_1)(W_0 - C_0)(1+R)] = (W_0 - C_0)U'(C_0)\iota$$

or

$$E[M(1+R)] = \iota,\ \ E[R - rf] = -\text{cov}(M,R)/E(M)$$

with

$$M = U'(C_1)/U'(C_0)$$

b. Remember: if X~N(m,s^2), then E[exp(X)] = exp(m+0.5s^2). Assume the stochastic discount factor M is lognormally distributed. Also the risky market return 1+R is lognormally distributed. In particular,
ln(1+R) ~ N(0.08, 0.04), ln(M) ~ N(-0.02,0.0001), and
correlation(ln(1+R),ln(M))= -25%.
What is the riskfree rate in equilibrium?

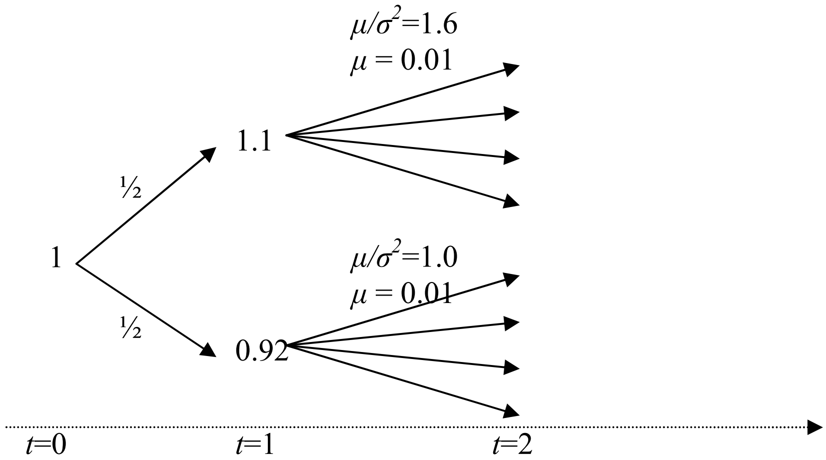
$$(1+rf)^{-1} = E[M] = E[\exp(\ln(M))] = \exp(-0.02 + 0.5 * 0.0001) \rightarrow rf \approx 2\%$$

c. Assuming your previous answer is correct, does the FAPE hold for the market return? What should your action now be if you believe that the pricing model is correct?

E[M (1+R)] = exp(0.08 – 0.02 + 0.5 * (0.04 + 0.0001 – 0.5*(0.04*0.0001)$^{1/2}$)) > 1, so does NOT hold. If the model is correct, the expected return exceeds the equilibrium return. So buy the risky asset!.

[CONTINUE WITH PART (d) ON NEXT PAGE]

d. A stock price has the following properties over the first and second period.



The riskfree rate is 0%. Probability of +10% or -8% at t=1 are ½. The reward risk ratio (i.e. expected *return* of the risky asset divided by *return* variance) over the second period (from t=1 to t=2) is given by μ/σ^2 and differs after the positive and negative return over the first period, respectively. The expected return is 1% per period, throughout.

The investor is a mean-variance optimizer with risk aversion parameter 2, so max $EW_2 - 2var(W_2)$, where W_2 is end-of-period-2 wealth.

Assume the myopic asset demand for a one-period investor is 30.5% in the risky asset.

Derive the risky asset demand for a two-period investor that can (dynamically) rebalance at t=1. [the final answer need not be a number, but can also be a formula]

in the up-state, standard MV analysis results in the allocation $1.6/(4W_1)$ to the risky asset. In the down state $1.0/(4W_1)$

This means for the first period (define g (k) as the random variable indicating 0.4 (0.1) in the upstate, and 0.25 (-0.08) in the downstate;):

max E[W_1*(1 + g*R2/W_1)] – 2 var(W_1 + g*R2) -->

max E[W_1 + g*mu2] – 2 [var(W_1) + var(g*mu2) + 2 cov(W_1,g*mu2)] -->

FOC

mu1*W_0 – 2 (2*alpha*W_0^2 sigma1^2 + 2 W_0 cov(k,g*mu2)

alpha = (mu1 - 4 cov(k,g*mu2)) / (4*W_0 sigma1^2) < mu1 / (4*W_0 sigma1^2)

e. Is there a positive or a negative hedging demand? Explain the intuition.

This is a mean variance investor. If the return over the first period is positive (negative), the reward/risk ratio over the subsequent period increases (decreases). For a long-term investor, the asset is thus even riskier. Therefore, hedging demand is negative (in fact, the asset does increase reinvestment risk: bad return is followed by a worsining of the subsequent IOS)

Q4:

a. Give the definition of Value-at-Risk and of Conditional-Value-at-Risk.

see book

b. Explain two drawbacks of VaR.

does not say anything on tail behavior beyond the VaR; is not subadditive, so does not inuitively account for diversification benefits; gives rise to degenerate portfolios (gambles) if derivatives are allowed.

c. Interest rate changes are normally distributed, with mean 2 basis points and standard deviation 10 basis points over the coming period. You hold a callable bond, priced currently at €915, with modified duration of 3 years. What is the 95% VaR on your bond position approximately (in €)?
[If X is normally distributed, P[$X < -1.645$] = 5%.]

Delta VaR using modified duration:
price change is approximately -3*915*yield change
So 95% worst likely yield change is 2bp + 1.645*10bp, so
VaR = 3 * 915 * (2 + 16.45) / 10000 = 5.06

d. Given the world with 50 possible states (all with equal probability of 2%) and Arrow-Debreu securities with prices as in the following table. Assume you have €10,000 and you want to maximize expected return, given that your 90% VaR is zero, and given you cannot take short positions.
Write down the formal optimization problem you are solving.

q_x *is the quantity of state (AB security) x*

p_x *is the price of state (AB security) x*

$q_{1:50} < q_{2:50} < \ldots < q_{50:50}$ *the sorted quantities*

$$\max \sum_{x=1}^{50} q_x / 50$$

st

$$q_x \geq 0$$

$$\sum_{x=1}^{50} q_x p_x \leq 10{,}000$$

$$q_{6:50} \geq 10{,}000$$

e. Derive your optimal portfolio. [you may buy fractions of the Arrow-Debreu securities, but you cannot take short positions.]

Buy 10000 of x=1 ... 45, costs 9790 – 1720 = 8070. Left with 1930, buy 1,930,000 additional x=1. VaR = 0, expected return = 0.88*0 + 0.02*1930% + 0.1*(-100%) = 376%

f. How is your answer affected if you can take short positions in the 15th and 35th asset.

If you can take short positions, you will create a degenerate portfolio: 10,000 long in everything except e.g. 35, invest the rest in x=1, and then each time short x=35 and buy 26 times x=1, ad inifinitum. VaR = 0, expected return is infinite. [expected loss also by the way!!]

Prices of Arrow-Debreu securities (pay out 1 Euro in state *x* only, and 0 otherwise)

x	price	*x*	price	*x*	price	*x*	price	*x*	price
1	€ 0.001	11	€ 0.012	21	€ 0.017	31	€ 0.023	41	€ 0.029
2	€ 0.002	12	€ 0.013	22	€ 0.017	32	€ 0.023	42	€ 0.030
3	€ 0.002	13	€ 0.014	23	€ 0.017	33	€ 0.024	43	€ 0.031
4	€ 0.002	14	€ 0.014	24	€ 0.018	34	€ 0.026	44	€ 0.031
5	€ 0.004	15	€ 0.015	25	€ 0.018	35	€ 0.026	45	€ 0.031
6	€ 0.007	16	€ 0.016	26	€ 0.019	36	€ 0.027	46	€ 0.033
7	€ 0.007	17	€ 0.017	27	€ 0.020	37	€ 0.027	47	€ 0.034
8	€ 0.008	18	€ 0.017	28	€ 0.020	38	€ 0.028	48	€ 0.035
9	€ 0.010	19	€ 0.017	29	€ 0.021	39	€ 0.028	49	€ 0.035
10	€ 0.011	20	€ 0.017	30	€ 0.022	40	€ 0.028	50	€ 0.035

Sum of all the prices is €0.979.