Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration

Programs:	M.Sc. Finance, M.Sc. Quantitative Finance
Exam:	Investments 4.1
Course code:	60412040
Date:	Oct 26, 2007
Time:	12:00 – 14:45
Duration:	2 hours, 45 minutes
Parts:	The exam has 4 questions, 17 subquestions.
Grading:	Each of 17 subquestions in the exam will be equally weighted. Perhaps redundantly: the written exam makes up 70% of your final grade. The remaining 30% is scored by the cases. The exam can be retaken. The cases cannot.
Results:	Results will be made known as soon as possible, but at the latest Friday, Nov 9, 2007.
Inspection:	You can inspect your marked exam papers Tuesday, Nov 13, 9:00am. The room will be announced via the monitor system.
Remark:	Provide complete answers (including computations where appropriate). Always provide motivation/explanation of your answer, even if this is not mentioned explicitly in the question. A short ‘yes’ or ‘no’ will never do as an answer. But also be concise/crisp in your answer, or it will take you too much time to write it down. Use your time efficiently.

Scan for the (in your opinion) easier questions first.
Good luck!

This document has 6 pages (this page included)

If X~N(m,s^2), then E[exp(X)] = E[e^X] = exp(m + 0.5s^2).

If X~N(0,s^2), then E[X^3] = 0.

If $X \sim \mathrm{N}(\mu, V)$, then $w'X \sim \mathrm{N}(w'\mu, w'Vw)$.

E[XY] = cov(X,Y) + E[X]E[Y], cov(X,Y) = E[XY] - E[X]E[Y],
E[X^2] = var(X) + E[X]^2, var(X) = E[X^2] - E[X]^2,

$X = \exp(\ln(X))$; $\exp(X)\exp(Y) = \exp(X+Y)$; $\ln(X^a) = a\ln(X)$

$$\frac{\partial \exp(x)}{\partial x} = \exp(x); \quad \frac{\partial x^a}{\partial x} = a \cdot x^{a-1}; \quad \frac{\partial a^x}{\partial x} = a^x \cdot \ln(a);$$

$$\frac{\partial \ln(x)}{\partial x} = x^{-1}; \quad \frac{\partial (c \cdot x + b)^a}{\partial x} = c \cdot a \cdot (c \cdot x + b)^{a-1}$$

[new, but known]

QUESTION 1

1a.
State the Fundamental Asset Pricing Equation (FAPE) in its two basic forms.
Also verbally explain the underlying economic interpretation/mechanism of the FAPE (why are low correlations rewarded, or not?).
Finally, indicate how the FAPE may be used empirically to assess the performance/track record of a specific Mutual Fund.

$$\mathrm{E}[M \cdot (1+R)] = 1; \quad \mathrm{E}[R^e] = \frac{-\mathrm{cov}[M, R^e]}{\mathrm{E}[M]}$$

High expected returns hedge against marginal utility risk. If returns are high when consumption is low (marginal utility is high) these assets are desirable for consumption smoothing. In equilibrium there demand is stronger, driving up the price and therefore driving down the expected returns.

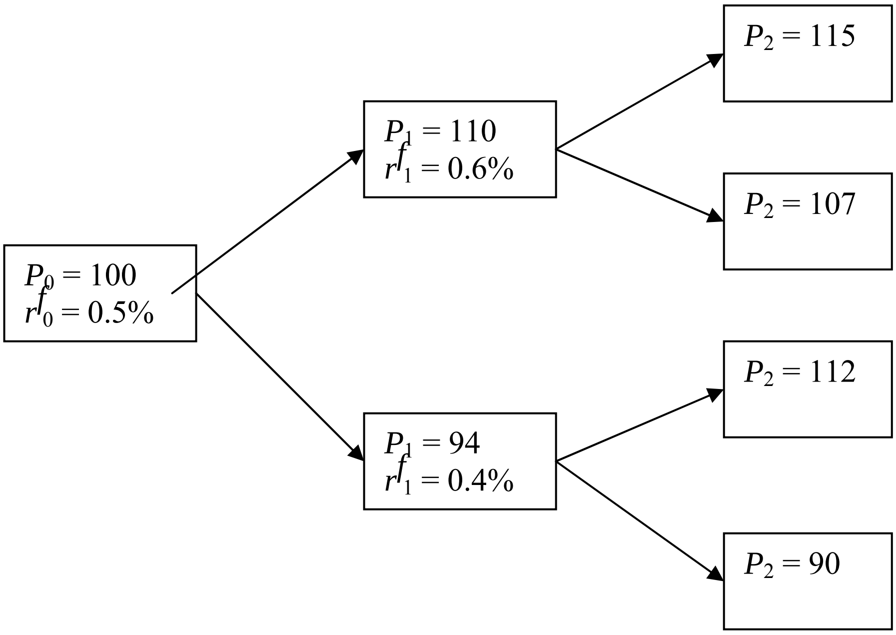
Empirically for given M, test whether average return of the MF exceeds – cov of M with MF return divided by expectation of M. If so, the MF generates alpha compared to the pricing model M.

1b.
Show that if the SDF is linear in the market return, the CAPM holds.
Also indicate under what conditions on utility and return distributions the SDF will be linear in the market return.

See book and slides.
Conditions: e.g., quadratic utility and homogeneous agents.

1c.

Consider the following simple world (probability of up/down is 50% always):

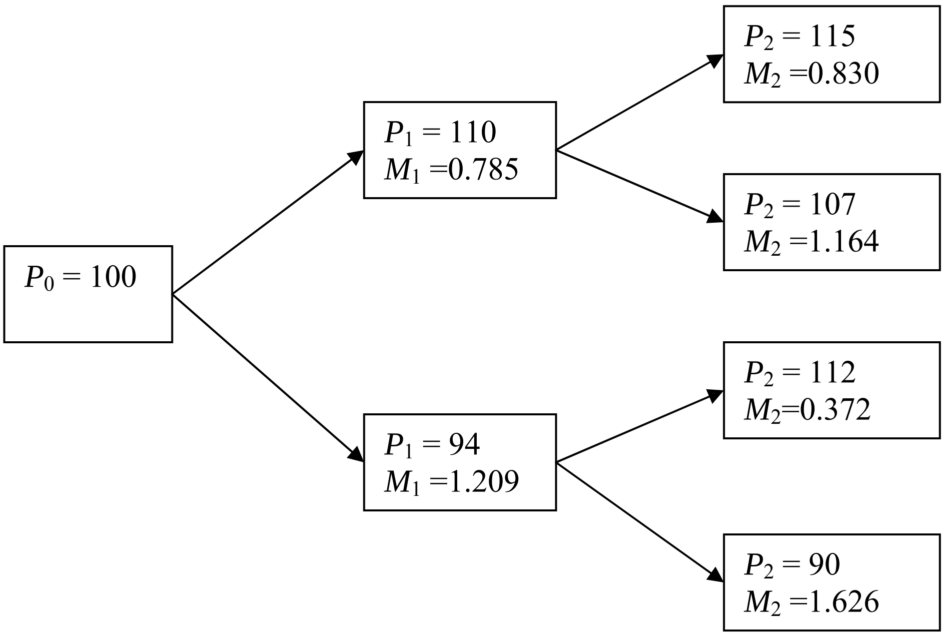


Here, P_t is the price of a risky security, and r^f_t is the risk-free rate applicable over the subsequent period.
Compute the SDF at t=1 and t=2 in all states (3 digits required).

M^{du}=0.396233249
M^{dd}=1.595798624
M^{uu}=0.909542744
M^{ud}=1.078528827
M^{u} =0.808457711
M^{d} =1.18159204

Solve directly from the pricing equations for the two assets, e.g.
$100 = ½\ 110.0\ M^u + ½\ \ 94.0M^d$
$100 = ½\ 100.5\ M^u + ½\ 100.5M^d$

1d.
Consider the following simple world (probability of up/down is 50% always):



Here, P_t is the price of a risky security, and M_t is stochastic discount factor (SDF).

What is the price at t=0 of a European at-the-money call option that matures at t=2?
Also, what is the price of a European at-the-money put?
Finally, verify whether put-call parity holds, i.e., security+put = call + present value strike (in cents.

Put = 4.914585 = $E[M_1 M_2 (100-P_2)^+]$
Call = 5.3916015 = $E[M_1 M_2 (P_2-100)^+]$
Present value of strike (100) = 99.5218 = $E[M_1 M_2]$
100 + 4.914585 = 5.3916015+99.5218 = 104.9134015
So parity holds (approximately).

QUESTION 2

2a.
State the EMH and give 4 ways to empirically test it.

see book and slides:
variance ratios
regression tests
fama-macbeth
multiple-equations tests with prices and dividends.
Panel tests with restrictions on intercepts and demeaned risk factors.

2b.
You have the 2 factor model

$$R_{it} - r^f = \alpha_i + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + e_{it} = \alpha + \beta_i' f_t + e_{it}.$$

Factor 1 has mean 2% and standard deviation 8%, factor 2 has mean 4% and standard deviation 15%. The factors are uncorrelated. The following numbers hold.

Stock type	α_i	β_{i1}	β_{i2}	stdev(e_{it})
1	-1%	1.00	-2.00	12%
2	2%	-0.50	0.50	15%
3	4%	1.00	1.00	18%

Build a long-only portfolio with positive alpha but zero systematic risk.
What does the portfolio look like?

1a-0.5b+1(1-a-b) = 0
-2a+0.5b+1(1-a-b) = 0
→ a = 2/9, b=2/3, c=1-a-b=1/9.
Alpha = -2/9 + 4/3+4/9=14/9=1.56%

2c.
Describe the Fama-MacBeth procedure to test whether your long-short portfolio from question (b) really has alpha, or whether this is beta risk after all.

See book and slides.

2d.
Explain in what way your conclusion in (c) is sensitive to the pricing model you have adopted and how you can address this sensitivity issue in empirical research.

If you use the wrong pricing model, you may find spurious alpha. For example, if you omitted a risk factor. In empirical research, this issue can be partially addressed by investigating the stability of the estimates, the premia, or by investigating the robustness of the alpha by considering alternative, more elaborate pricing models.

QUESTION 3

3a.
Give a brief description in words of the equity premium puzzle and the the riskfree rate puzzle.
Also discuss at least two possible solutions that have been proposed to solve the EPP and their success/failure as a solution.

See book and slides

3b.
For an agent with utilty function

$$U(W) = -(W+1)^{-3},$$

give the relative and absolute risk aversion coefficient.
Also give the amount the agent is willing to pay to avoid a fair gamble between a wealth level of 90 (with 50%) or 110 (with 50%).
[hint: you have to find this premium numerically using your calculator]

$$U' = 3(W+1)^{-4},\ U'' = -12(W+1)^{-5},$$

$$RA = 4(W+1)^{-1}$$

$$RR = 4W(W+1)^{-1}$$

$$U(90) = -1.3270*10^{-6}$$

$$U(110) = -7.3119*10^{-7}$$

$$\text{Average}: \left(-1.3270*10^{-6} - 7.3119*10^{-7}\right)/2 = -1.0291*10^{-6}$$

$$U^{-1}(u) = (-u)^{-1/3} - 1,\ \text{so}\ U^{-1}(-1.0291*10^{-6}) \approx 98, \text{so premium of 2}$$

3c.
We have an agent who consumes at time t=0,1,2,3. At t=3, she consumes everything she has. From consumption at the different times, she derives utility.
The economy this agent lives in, has two risky asset and a riskfree asset. The riskfree asset pays a one-period fixed return.
The agent cannot take short positions in any of the three assets, and can invest a maximum of 50% of her investable wealth in any of the risky assets at any time.
Expected returns depend on the current level of the aggregate dividend-yield in this economy.
The agent wants maximum expected utility over her life-time.

State (in formulas) the maximization problem of this agent (objective and constraints, optimizing variables, state dependencies, etc).
Also indicate how this problem can be solved, and on what variables the solution will depend.
[note: you do not actually need to solve it, but be accurate in your description of how to solve it]

$$\max \ \mathrm{E}\left[\sum_{t=0}^{3} \theta^t \cdot U(C_t)\right],$$
$$W_{t+1} = (W_t - C_t)\cdot R^*_{t+1}, \quad t = 0,1,2$$
$$C_3 = W_3$$
$$0 \le x_{1t}, x_{2t} \le 0.5, \ t = 0,1,2$$
$$1 - x_{1t} - x_{2t} \ge 0, \ t = 0,1,2$$
$$x_{it} = x_{it}(W_t, dy_t), \ t = 0,1,2$$
$$C_t = C_t(W_t, dy_t), \ t = 0,1,2$$
$$R^*_{t+1} = 1 + r_t^f + x_{1t}(R_{1,t+1} - r_t^f) + x_{2t}(R_{2,t+1} - r_t^f), \ t = 0,1,2$$

3d.
Barberis argues that stocks hedge against changes in the investment opportunity sets, and therefore have a positive hedging demand compared to myopic investors.
What does Barberis mean? Discuss why it is relevant.

See paper: the assets take a larger fraction in the mix compared to myopic allocation, because the assets hedge reinvestment risk. For example, the assets have high expected returns after poor realized returns. This property helps long term investors to stabilize consumption paths (consumption smoothing), such that long term investors put more of these assets into their portfolio (compared to myopic investors). Relevance: personal financial planning or pension fund context.

3e.
Consider the model for log dividend yields dy_t and continuously compounded holding period returns h_{t+1},

$$\begin{pmatrix} h_{t+1} \\ dy_{t+1} \end{pmatrix} = \begin{pmatrix} -0.025 \\ 0.0015 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0.95 \end{pmatrix}\begin{pmatrix} h_{t+1} \\ dy_{t+1} \end{pmatrix} + \begin{pmatrix} u_{t+1} \\ w_{t+1} \end{pmatrix},$$

$$\begin{pmatrix} u_{t+1} \\ w_{t+1} \end{pmatrix} \sim \mathrm{N}\left(0, \begin{pmatrix} 0.19\% & -0.0062\% \\ -0.0062\% & 0.00026\% \end{pmatrix}\right).$$

Assume that the average log-dividend price ratio is given by $\rho = 0.99$, and the (unconditional) variance of h_{t+1} is 0.20%. What proportion of the variance of stock returns is explained by discount rate news? Explain for your result.

[Hint: for a 2 x 2 matrix, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} d/k & -b/k \\ -c/k & a/k \end{pmatrix}$, with $k = ad - bc$.]

$$\rho A = \begin{pmatrix} 0 & 0.99 \\ 0 & 0.9405 \end{pmatrix}, \ (I - \rho A)^{-1} = \tfrac{1}{0.0595}\begin{pmatrix} 0.0595 & 0.99 \\ 0 & 1 \end{pmatrix},$$

$$\lambda' = e_1' \ \rho A (I - \rho A)^{-1} = \begin{pmatrix} 0 & \tfrac{0.99}{0.0595} \end{pmatrix},$$

$$\text{var}(DR) = \left(\tfrac{0.99}{0.0595}\right)^2 0.00026\% \rightarrow \text{var}(DR) / \text{var}(h) = 36\%.$$

Intuition : expected returns are sticky, so a small stock affects many future discount rates and therefore has a leveraged effect on direct stock returns.

QUESTION 4

4a.
Define VaR.
Define the Delta-VaR approach.
For two normally distributed risky returns with means 10% and 15%, standard deviations 10% and 20%, and correlation 30%, compute the 99%-VaR of a portfolio of €10 milion that has a position of 35% in security 1, and 65% in security 2.
[P(Normal(0,1) < -2.326) = 1%]

See book and slides.
$10M * (2.326*(0.35^2*0.1^2 + 0.65^2*0.2^2 + 2*0.3*0.35*0.65*0.2*0.1)^{1/2} – 0.35*0.1 – 0.65*0.15) = 2,034,037$

4b.
A three-year bond (par value €1000) is issued currently at par. It pays an annual coupon of 5%. The current forward rate curve is $f_{1,t=0}$=4%, $f_{2,t=0}$=5%, $f_{3,t=0}$=6.11% for the first 3 years, such that
1000 = 50/1.04 + 50/(1.04*1.05) + 1050/(1.04*1.05*1.0611).
We assume that changes in the forward rates are normally distributed,

$$\begin{pmatrix} f_{1,t=1} \\ f_{2,t=1} \end{pmatrix} \sim N\left(\begin{pmatrix} f_{1,t=0} \\ f_{2,t=0} \end{pmatrix}, \begin{pmatrix} 0.0025\% & 0.00225\% \\ 0.00225\% & 0.0025\% \end{pmatrix} \right).$$

Use the Delta-method to compute the approximate 99%-VaR of a long position in the bond over a one-year holding period.
[Hint1: P(Normal(0,1) < -2.326) = 1%]
[Hint2: write down the price at t=1 just before coupon payment, and approximate it with a (bivariate) first order Taylor series approximation.]

At t=1, the bonds value is
50 + 50/(1+f1) + 1050/((1+f1)(1+f2)) = 1059.615
Derivative wrt f2:
$-1050/((1+f1)(1+f2)^2) = 915.75$
Derivative wrt f1
$-50/(1+f1)^2 - 1050/((1+f1)^2(1+f2)) = 970.78$
So price at t=1 is approximately:
$1059.615 - 970.78*(f_{1,t=1} – f_{1,t=0}) - 915.75*(f_{2,t=1} – f_{2,t=0}) = 1059.615 + x$,
where x is normally distributed with mean zero and standard deviation
$(970.78^2\ 0.0025\% + 915.75^2\ 0.0025\% + 2*0.00225\%*970.78*915.75)^{1/2} = 84.53$
Therefore the VaR is approximately
1000 – (1059.615 – 2.326*84.53) = 137.00

4c.
Given a set of 2,500 observed daily returns on a portfolio, you can compute the VaR of this portfolio. You want to construct a 95% confidence interval for your estimate of the VaR. Describe a bootstrap procedure to do this.

Take returns → bootstrap a sample of same historical length
On the bootstrapped sample, compute the VaR same way as for the empirical sample.

Retain the VaR value.
Repeat the above 3 steps many times.
Rank the bootstrapped VaR values, throw away the top and bottom 2.5%. Boundaries form the confidence interval.

4d.
Describe the Casino effect for Mean-VaR optimization (the Vorst paper). Also indicate in what way the casino effect is relevant for investors that are restricted from using options directly in their asset mix.

See paper and slides.
Importance: also if you cannot use options directly, but you can dynamically rebalance, you can approximate option pay-offs by dynamic asset allocation.