This document contains 6 trial exam questions.
Some questions are too easy for the real exam, some (by contrast) are more difficult.
This set of questions is too long for a real exam of 2h45min.

Standard results:

If $-1 < a < 1$, then $1 + a + a^2 + a^3 + a^4 + \ldots = (1-a)^{-1}$.

If $-1 < a < 1$ and $y_t = m + a \cdot y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim \mathrm{N}(0, \sigma^2)$, then $\mathrm{E}_t[y_t] = m + a \cdot y_{t-1}$ and $\mathrm{E}[y_t] = m/(1-a)$.

If $\varepsilon_t \sim \mathrm{N}(\mu, \sigma^2)$, then $\mathrm{E}[\exp(\varepsilon_t)] = \mathrm{E}[e^{\varepsilon t}] = e^{\mu + \frac{1}{2}\sigma^2} = \exp(\mu + \frac{1}{2}\sigma^2)$.

TRIAL QUESTION 1.
Consider a two-period model

$$\max_{C_t,\alpha_t} U(C_t) + \vartheta U(C_{t+1}),$$
$$C_{t+1} = (W_0 - C_t)\cdot \alpha_t (1 + R_{t+1}),$$

where ϑ is the rate of time preference parameter, C_t is consumption at time *t,* α_t is the vector of asset allocation weights (in fractions), and R_{t+1} is the return over the period [t,t+1]. We assume $\vartheta = 0.9994$ and

(1) $$\begin{pmatrix} c \\ r \end{pmatrix} = \begin{pmatrix} \ln(C_{t+1}/C_t) \\ \ln(1+R_{t+1}) \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 3\% \\ m \end{pmatrix}, \begin{pmatrix} 0.2\% & 0.2\% \\ 0.2\% & 4\% \end{pmatrix} \right).$$

Furthermore, utility is specified as $U(C) = \ln(C)$.

a) Show that this utility function has a constant relative risk aversion coefficient equal to 1.

b) Compute the level of the riskfree rate at *t* for the period (*t,t*+1).

c) Explain what is meant by the risk-free rate puzzle and by the equity premium puzzle.

d) State the fundamental asset pricing equation (FAPE) using the stochastic discount factor (SDF) M_{t+1}. Use the expression of the FAPE with excess returns on individual assets on the left hand side of the equation.

e) Pick an asset that is positively correlated with the SDF. Should this asset have a positive or negative risk premium? Explain and provide the intuition.

f) Assume your answer to (b) would be 3%, and assume $\mathrm{cov}_t\left[M_{t+1}, R_{m,t+1}\right] = 8\%$,what would be the equilibrium market risk premium?

g[difficult]) Assume your answer to (b) would be 3%, and assume the log gross return on the market portfolio had a covariance with log gross consumption growth as given in equation (1) above. What would be the equilibrium market risk premium?
[tip: equate the straightforward expression for the expected excess return to that following from the FAPE in order to compute *m*.]

TRIAL QUESTION 2.

Assume the riskfree rate is 3%, and the market price of risk $\lambda = (\mu_t - r_t)/\sigma_t^2$ is constant and equal to 1.5. Assume that returns follow the following process,

$$R_{t+1} - 0.03 = \lambda\sigma_t^2 + \sigma_t\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathrm{N}(0,1),$$
$$\sigma_t^2 = 0.004 - 0.9\cdot\left(R_t - 0.03 - \lambda\sigma_{t-1}^2\right)^2, \; \sigma_0^2 = 0.04.$$

a) Explain the properties of the above process for the returns. In particular, what stylized empirical fact in financial markets does the 2nd equation try to pick up, and how does it do that?

b) What is the conditional (at time *t*) expected excess return?

c[difficult]) What is the unconditional expected excess return on the market? At what volatility levels do the conditional and unconditional expected return coincide?

d) Assume we perform an OLS regression of excess returns on lagged excess returns and a constant. If we found that this OLS coefficient for lagged returns would be unequal to zero (as we actually would, in large samples), would we have return predictability?
Argue that this does not mean that the efficient markets hypothesis (EMH) is violated. Also provide a framework where such predictability (non-zero OLS coefficient) would imply that the EMH is violated.

f) Explain the EMH in its 3 different forms. Which form would be violated in the second part of question (e)?

TRIAL QUESTION 3.
You are a mean-variance (MV) investor with objective

$$\max \mu_p - 3\sigma_p^2.$$

You can choose the riskfree asset, yielding 3%, or a risky asset with mean μ and variance σ^2. Using past data, you have estimated the mean to be 8%, and the variance to be 0.0225. You currently have €100,000.

a) Derive your optimal fraction in the risky asset.

b) Now assume your mean of 8% is based on a sample of historical returns. In particular, your estimate $\hat{\mu}$ given your sample has distribution N(0.08,0.0009). If you account for this parameter uncertainty, how will your allocation change?

c) Now assume that the estimated expected return comes from a forecasting equation

$$R_{t+1} = \hat{a} + \hat{b} DY_t + \varepsilon_{t+1} = 0.03 + 1 \cdot DY_t + \varepsilon_{t+1},$$

with time t dividend yields equal to 0.05. Again, these regression parameters have been estimated and

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0.03 \\ 1.00 \end{pmatrix}, \begin{pmatrix} 0.0001 & 0.001 \\ 0.001 & 0.04 \end{pmatrix} \right).$$

If you do not account for parameter uncertainty, what is your optimal allocation?

d) If you account for the parameter uncertainty in (c), what is your optimal allocation?

e) Show how your answer to (d) varies if you observe higher or lower current dividend yields. [be careful!] Compute the difference between accounting for and not accounting for parameter uncertainty for DY=2% and DY=10%.

f) Carefully describe a bootstrap procedure to account for parameter uncertainty in the above case with return predictability based on a regression equation with dividend yields. In your bootstrap procedure, also account for uncertainty in the estimated variance of the return (now set at 0.0225).

g) Explain 3 different methods to test for mutual fund persistence. Review the main findings in the literature here.

h) Carefullly explain the Fama-McBeth procedure for testing the CAPM (or APT).

TRIAL QUESTION 4.

Consider the standard Fama-French model,

(1) $R_{it} - r_t = \alpha_i + \beta_i (R_{mt} - r_t) + \gamma_i SMB_t + \delta_i HML_t + e_{it}$.

a) Explain how to construct SMB_t using quartile-based portfolios. (be clear!)

b) Assume the error terms e_{it} are uncorrelated (both time-series wise and cross-sectionally. You have a universe of 4000 stocks with the following properties.

Stock type	number of stocks of this type	α_i	β_i	γ_i	δ_i
A	1000	0	1.0	0.0	0.0
B	1000	0	0.5	1.0	0.0
C	1000	0	1.5	-0.5	-0.5
D	1000	0	1.0	0.0	-1.0

Further, all idiosyncratic variances are equal to $\sigma_i^2 = 0.01$. The factors satisfy

$$\begin{pmatrix} R_{mt} - r_t \\ SMB_t \\ HML_t \end{pmatrix} \sim \mathrm{N}(\mu_F, V_F) = \mathrm{N}\left(\begin{pmatrix} 4\% \\ 2\% \\ 3\% \end{pmatrix}, \begin{pmatrix} 4\% & 1\% & 1\% \\ 1\% & 2\% & -0.5\% \\ 1\% & -0.5\% & 1.5\% \end{pmatrix} \right)$$

Assume you hold a portfolio of 1M with *all* the A-type stocks. What is the percentage of this portfolio's return variance that is systematic? (2 digits)

c) What if you only held 10 A-type stocks in your 1M portfolio (and no other stocks). How would your answer to (b) change?

d) Also compute the VaR of the portfolio in (c) and its systematic VaR. (2 digits) Compare the percentage contribution of the systematic VaR to the total VaR.

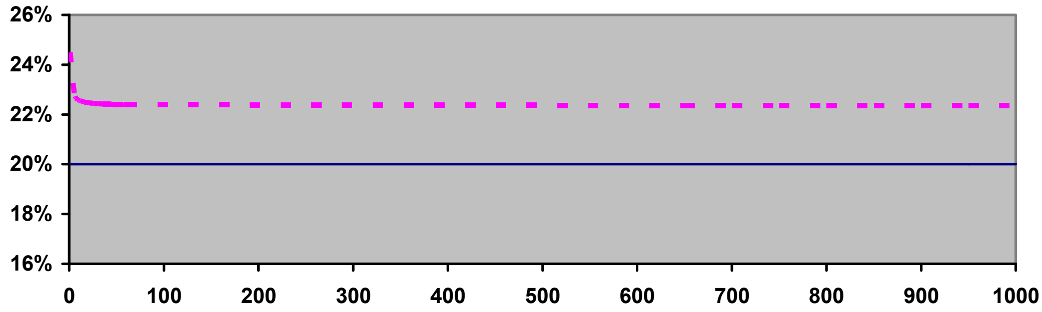
e) Explain why the answers to (c) and (d) are or are not different.

f) Assume that instead as mentioned in the above table, $\alpha_C = 0.02 \neq 0$. What does this mean/imply? Explain some possible causes for $\alpha_C > 0$.

g) Construct a zero-cost portfolio of stocks to exploit the positive alpha in (f) without incurring systematic risk exposure. (Assume you can go long and short in stocks). What are the risk (standard deviation) and expected payoff of this portfolio?

h) What is the number of available C-type stocks were 10 instead of 1000. What would change to your answer in (g)?

i) Assume you are given model (1) above and the table in part (b). Consider you build a portfolio of A-type stocks only. **But** when you make a graph of the portfolio standard deviation against the number of stocks included in your portfolio, the graph looks something like this. (dashed pink curve)



First explain the graph. Second, discuss what the graph tells about the entries in line A of the table in part (b) or about model (1) above.

TRIAL QUESTION 5.

Assume there are only 10 possible realizations of the stock return, namely -10%, -7%, -4%, -1%, 2%, 5%, 8%, 11%, 14%, 17%. All outcomes have equal probability. Also assume you have 10 digital options (are Arrow-Debreu securities). Each of these options pays out an amount of $1 in one particular state of the world only. So one option pays $1 if and only if the realized return is -10%, another option pays $1 if and only if the realized return is -7%, and so on. Denote the prices of the options by p_i.

a) State the fundamental asset pricing equation using a stochastic discount factor (SDF). From that, derive an expression for the SDF in terms of the option prices only.

b) Assume you have a utility function of the following type

$$(1) \qquad U(W) = \begin{cases} W & \text{if } W \geq A, \\ 3W - 2A & \text{if } W < A. \end{cases}$$

Let A=W_0=100. Sketch the definition of loss aversion and show how this utility function satisfies this definition.

c[difficult]) Assume you have the following price table.

Stock Return	-10%	-7%	-4%	-1%	2%	5%	8%	11%	14%	17%
Probability	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Price	1.11	1.08	1.04	1.01	0.98	0.95	0.93	0.90	0.88	0.85

Show why the optimal portfolio if you are an expected utility optimizer with utility as given in (1), has a short position in the -10%-option, and positions of 100 in all the others.
[tip: if you had 200 options for state -1%, and zero for all the others, what is an easy way to improve your utility the most?]

d[difficult]) What would change if you had A=10 and still W_0=100?

e) What is the riskfree rate implied by the table in part (c)?

f) How would you implement your solution to (c) and (d) if only plain vanilla options were available in the market.

g) What would you do if you can do dynamic rebalancing of your asset portfolio, but you can only invest in stocks and a riskfree asset each time (so not in the digital or plain vanilla options)?

h) Consider a call option entitling you to buy 100 stocks at the current stock price at the same moment the digital options expire. Approximate the price of this option given the prices of the digital options in the table above. Provide the details of how you set up your computations.

i) Discuss the behavioral biases of loss aversion, ambiguity aversion, and over-confidence.

j) What are the differences between expected utility theory and prospect theory. Discuss when and why you should use either of these theories.

TRIAL QUESTION 6.

Consider the following simple world, where prices of a risky asset are given by

t=0		t=1		t=2	
100	up	116	up-up	119	
			up-down	115	
	down	90	down-up	115	
			down-down	85	

All ups and downs happen with equal probability of 50%. The one-period riskfree rate is 1% at all times. Your utility function is given by

(1) $U(W) = -0.25W^{-4}$.

a) Compute the one-period stochastic discount factor (SDF) at time 0, at time 1 in the up-state, at time 1 in the down-state.

b) Use the SDF to price an at the money call option and an at the money put option, both maturing at time t=2. Does put-call-parity hold?

c) Show whether -94.04% is the optimal fraction invested in the risky asset at time t=1 if you are in the up-state?

d) Similar for 92.57% in the down-state?

e) Show that the optimal fraction invested in the risky asset at time t=0 if you are a one-period expected utility maximizer is 24.20%? [so a short-horizon investor]

f) Your optimal fraction invested in the risky asset at time t=0 if you are a two-period expected utility maximizer is 36.90%. [so a long-horizon investor]
Explain why this is different from (e).

h) Explain why in a long-horizon investor problem the investor may or may not hold a larger fraction of risky assets if (s)he is younger. (sketch some model assumptions that would be needed to obtain such a result)