This document contains 6 trial exam questions.
Some questions are too easy for the real exam, some (by contrast) are more difficult.
This set of questions is too long for a real exam of 2h45min.

Standard results:

If $-1 < a < 1$, then $1 + a + a^2 + a^3 + a^4 + \ldots = (1-a)^{-1}$.

If $-1 < a < 1$ and $y_t = m + a \cdot y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim \mathrm{N}(0, \sigma^2)$, then $\mathrm{E}_t[y_t] = m + a \cdot y_{t-1}$ and $\mathrm{E}[y_t] = m/(1-a)$.

If $\varepsilon_t \sim \mathrm{N}(\mu, \sigma^2)$, then $\mathrm{E}[\exp(\varepsilon_t)] = \mathrm{E}[e^{\varepsilon t}] = e^{\mu + \frac{1}{2}\sigma^2} = \exp(\mu + \frac{1}{2}\sigma^2)$.

TRIAL QUESTION 1.
Consider a two-period model

$$\max_{C_t,\alpha_t} U(C_t) + \vartheta U(C_{t+1}),$$
$$C_{t+1} = (W_0 - C_t)\cdot \alpha_t (1 + R_{t+1}),$$

where ϑ is the rate of time preference parameter, C_t is consumption at time *t*, α_t is the vector of asset allocation weights (in fractions), and R_{t+1} is the return over the period [t,t+1]. We assume $\vartheta = 0.9994$ and

(1) $$\begin{pmatrix} c \\ r \end{pmatrix} = \begin{pmatrix} \ln(C_{t+1}/C_t) \\ \ln(1+R_{t+1}) \end{pmatrix} \sim N\left(\begin{pmatrix} 3\% \\ m \end{pmatrix}, \begin{pmatrix} 0.2\% & 0.2\% \\ 0.2\% & 4\% \end{pmatrix} \right).$$

Furthermore, utility is specified as $U(C) = \ln(C)$.

a) Show that this utility function has a constant relative risk aversion coefficient equal to 1.

$U' = 1/C,\ U'' = 1/C^2,\ U''\cdot C/U' = 1$, which is a constant.

b) Compute the level of the riskfree rate at *t* for the period (*t*,*t*+1).

$(1+r^f)^{-1} = E_t[M_{t+1}] = E_t[\vartheta U'(C_{t+1})/U'(C_t)] = E_t[\vartheta e^{-c}] =$
$0.9994\exp(-0.03 + \frac{1}{2}0.002) \Rightarrow r^f \approx 3\%$

c) Explain what is meant by the risk-free rate puzzle and by the equity premium puzzle.

d) State the fundamental asset pricing equation (FAPE) using the stochastic discount factor (SDF) M_{t+1}. Use the expression of the FAPE with excess returns on individual assets on the left hand side of the equation.

$E_t[R_{i,t+1} - r^f_{t+1}] = -\text{cov}_t\left[M_{t+1}, R_{i,t+1}\right]/E_t[M_{t+1}]$

e) Pick an asset that is positively correlated with the SDF. Should this asset have a positive or negative risk premium? Explain and provide the intuition.

f) Assume your answer to (b) would be 3%, and assume $\text{cov}_t\left[M_{t+1}, R_{m,t+1}\right] = 8\%$, what would be the equilibrium market risk premium?

$E_t[R_{i,t+1} - r^f_{t+1}] = -0.08 * 1.03 \approx 8.24\%$

g[difficult]) Assume your answer to (b) would be 3%, and assume the log gross return on the market portfolio had a covariance with log gross consumption growth as given in equation (1) above. What would be the equilibrium market risk premium?
[tip: equate the straightforward expression for the expected excess return to that following from the FAPE in order to compute *m*.]

Should be given by minus covariance multiplied by 1.03, see (d). So in this case

$\mathrm{E}_t\left[M_{t+1}(1+R_{mt+1})\right] = \mathrm{E}_t\left[\vartheta \exp(-c+r)\right] = 0.9994 \cdot \exp(-0.03 + m + \tfrac{1}{2}(0.002 - 2\cdot 0.002 + 0.04))$

$\mathrm{E}_t\left[1+R_{mt+1}\right] = \mathrm{E}_t\left[\exp(r)\right] = \exp(m + \tfrac{1}{2}0.04)$

$\mathrm{cov}_t\left[M_{t+1}, R_{mt+1}\right] = \mathrm{cov}_t\left[M_{t+1}, (1+R_{mt+1})\right] = \mathrm{E}_t\left[M_{t+1}(1+R_{mt+1})\right] - \mathrm{E}_t\left[1+R_{mt+1}\right]\mathrm{E}_t\left[M_{t+1}\right] =$

$0.9994 \cdot \exp(-0.03 + m + \tfrac{1}{2}(0.002 - 2\cdot 0.002 + 0.04)) - \exp(0.0367 + \tfrac{1}{2}0.04)/1.03 =$

$0.9994 \cdot \exp(-0.03 + m + \tfrac{1}{2}(0.002 + 0.04)) \cdot \left(\exp(-0.002) - 1\right)$

Dividing the negative of this covariance by the expected SDF gives

$\exp(m + \tfrac{1}{2}0.04) \cdot \left(1 - \exp(-0.002)\right).$

But in equilibrium, the excess return from the FAPE should equal the straightforward expression for the excess expected return, i.e., $\mathrm{E}_t\left[1+R_{mt+1} - 1.03\right] = \exp(m + \tfrac{1}{2}0.04) - 1.03,$

such that in equilibrium

$\exp(m + \tfrac{1}{2}0.04) \cdot \left(1 - \exp(-0.002)\right) = \exp(m + \tfrac{1}{2}0.04) - 1.03 \Leftrightarrow$

$m \approx 0.01156.$ Therefore, the risk premium is going to be

$\exp(0.01156 + \tfrac{1}{2}0.04) - 1.03 = 0.21\%.$ This illustrates the equity premium puzzle.

TRIAL QUESTION 2.

Assume the riskfree rate is 3%, and the market price of risk $\lambda = (\mu_t - r_t)/\sigma_t^2$ is constant and equal to 1.5. Assume that returns follow the following process,

$$R_{t+1} - 0.03 = \lambda\sigma_t^2 + \sigma_t\varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathrm{N}(0,1),$$
$$\sigma_t^2 = 0.004 - 0.9\cdot\left(R_t - 0.03 - \lambda\sigma_{t-1}^2\right)^2, \ \sigma_0^2 = 0.04.$$

a) Explain the properties of the above process for the returns. In particular, what stylized empirical fact in financial markets does the 2nd equation try to pick up, and how does it do that?

It tries to pick up volatility clustering: if volatility is high, it will remain high for some time, if it is low, it will remain low. It does achieve this because the next volatility depends on the squared realized return in deviation from its conditional mean (expectation). The latter will be large with high probability if past volatility was large, causing future volatility to be large as well, therefore causing volatility clustering.

b) What is the conditional (at time *t*) expected excess return?

$$\mathrm{E}_t\left[R_{t+1} - 0.03\right] = \lambda\sigma_t^2 = 1.5(0.004 - 0.9(R_t - 1.5(0.004 - 0.9(R_{t-1} - \ldots)^2)\ldots)^2)$$

c[difficult]) What is the unconditional expected excess return on the market? At what volatility levels do the conditional and unconditional expected return coincide?

$$\mathrm{E}\left[R_{t+1} - 0.03\right] = \mathrm{E}\left[\lambda\sigma_t^2\right] = 1.5\mathrm{E}\left[0.004 + 0.9\mathrm{E}_{t-1}\left[(R_t - 0.03 - \lambda\sigma_{t-1}^2)^2\right]\right] =$$
$$1.5\mathrm{E}\left[0.004 + 0.9\mathrm{E}_{t-1}\left[\sigma_{t-1}^2\varepsilon_t^2\right]\right] = 1.5\mathrm{E}\left[0.004 + 0.9\sigma_{t-1}^2\right] =$$
$$1.5\left(0.004 + 0.9\mathrm{E}\left[\sigma_{t-1}^2\right]\right) = 1.5\left(0.004 + 0.9\mathrm{E}\left[0.004 + 0.9\mathrm{E}_{t-2}\left[(R_{t-1} - 0.03 - \lambda\sigma_{t-2}^2)^2\right]\right]\right) =$$
$$1.5\left(0.004 + 0.9\cdot 0.004 + 0.9\mathrm{E}\left[0.9\mathrm{E}_{t-2}\left[\sigma_{t-2}^2\varepsilon_{t-1}^2\right]\right]\right) =$$
$$1.5\left(0.004 + 0.9\cdot 0.004 + 0.9^2\mathrm{E}\left[\sigma_{t-2}^2\right]\right) =$$
$$1.5\left(0.004 + 0.9\cdot 0.004 + 0.9^2\cdot 0.004 + 0.9^3\cdot 0.004 + 0.9^4\cdot 0.004 + \ldots\right) =$$
$$1.5\cdot 0.004/(1-0.9) = 0.06.$$

So if vol at 20% (variance at 4%), unconditionally expected return equal to conditionally expected return.

d) Assume we perform an OLS regression of excess returns on lagged excess returns and a constant. If we found that this OLS coefficient for lagged returns would be unequal to zero (as we actually would, in large samples), would we have return predictability?

Argue that this does not mean that the efficient markets hypothesis (EMH) is violated. Also provide a framework where such predictability (non-zero OLS coefficient) would imply that the EMH is violated.

There is return predictability in this case.
In principle, EMH is not violated. Would be violated if we found a positive OLS coefficient, but in our economic model excess returns did not vary over time (constant risk premia).
Here however we have time varying risk premia through the constant market price of risk and the time varying volatilities.

f) Explain the EMH in its 3 different forms. Which form would be violated in the second part of question (e)?

Weak form only. (only lagged price/return info)

TRIAL QUESTION 3.
You are a mean-variance (MV) investor with objective

$$\max \mu_p - 3\sigma_p^2.$$

You can choose the riskfree asset, yielding 3%, or a risky asset with mean μ and variance σ^2. Using past data, you have estimated the mean to be 8%, and the variance to be 0.0225. You currently have €100,000.

a) Derive your optimal fraction in the risky asset.

$$\mu_p = 0.03 + 0.05\alpha$$
$$\sigma_p^2 = 0.0225\alpha^2$$
$$0.05 - 6 * 0.0225 * \alpha = 0 \Leftrightarrow \alpha = 37\%$$

b) Now assume your mean of 8% is based on a sample of historical returns. In particular, your estimate $\hat{\mu}$ given your sample has distribution N(0.08,0.0009). If you account for this parameter uncertainty, how will your allocation change?

Future return is now the sum of two uncorrelated normals,
$$R_{t+1} = \hat{\mu} + \varepsilon_{t+1} = \mathrm{N}(0.08, 0.0009) + \mathrm{N}(0, 0.0225) = \mathrm{N}(0.08, 0.0234),$$
so the new risky allocation is $0.05/(6*0.0234) = 35.6\%$.

c) Now assume that the estimated expected return comes from a forecasting equation

$$R_{t+1} = \hat{a} + \hat{b}DY_t + \varepsilon_{t+1} = 0.03 + 1 \cdot DY_t + \varepsilon_{t+1},$$

with time *t* dividend yields equal to 0.05. Again, these regression parameters have been estimated and

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0.03 \\ 1.00 \end{pmatrix}, \begin{pmatrix} 0.0001 & 0.001 \\ 0.001 & 0.04 \end{pmatrix} \right).$$

If you do not account for parameter uncertainty, what is your optimal allocation?

Same as in (a).

d) If you account for the parameter uncertainty in (c), what is your optimal allocation?

Future return is now the sum of two uncorrelated normals,

$R_{t+1} = \hat{\mu} + \varepsilon_{t+1} = (\hat{a} + 0.05\hat{b}) + N(0, 0.0225) =$

$N(0.08, 0.0001 + 2*0.001*0.05 + 0.04*0.05^2) + N(0, 0.0225) =$

$N(0.08, 0.0228)$,

so the new risky allocation is $0.05/(6*0.0228) = 36.5\%$.

e) Show how your answer to (d) varies if you observe higher or lower current dividend yields. [be careful!] Compute the difference between accounting for and not accounting for parameter uncertainty for DY=2% and DY=10%.

Similar as (d), but expected return now changes as well,

$R_{t+1} = \hat{\mu} + \varepsilon_{t+1} = (\hat{a} + DY*\hat{b}) + N(0, 0.0225) =$

$N(0.03 + DY, 0.0001 + 2*0.001*DY + 0.04*DY^2) + N(0, 0.0225) =$

$N(0.03 + DY, 0.0226 + 0.002DY + 0.04DY^2)$,

so the new risky allocation is $DY/(6*(0.0226 + 0.002DY + 0.04DY^2))$

with parameter uncertainty, and $DY/(6*0.0225)$ without.

So for DY = 2%, with 14.7%, without 14.8%

For DY = 10%, with 71.8%, without 74.1%

f) Carefully describe a bootstrap procedure to account for parameter uncertainty in the above case with return predictability based on a regression equation with dividend yields. In your bootstrap procedure, also account for uncertainty in the estimated variance of the return (now set at 0.0225).

Step 1: estimate the model on the data.
Step 2: Given the parameters, compute the regression residuals $\hat{\varepsilon}_t$.
Step 3: draw/bootstrap from the original residuals $\hat{\varepsilon}_t$ (with replacement) T residuals, denoted $\hat{\varepsilon}_t^{(b)}, t = 1, \ldots, T$.
Step 4: use the bootstrapped residuals to construct bootstrapped returns, $R_{t+1}^{(b)} = a + bDY_t + \varepsilon_{t+1}^{(b)}$.
Step 5: with the bootstrapped returns, perform the regression $R_{t+1}^{(b)} = a^{(b)} + b^{(b)} DY_t + e_{t+1}^{(b)}$ to obtain estimates of the regression parameters, $\hat{a}^{(b)}$ and $\hat{b}^{(b)}$, and of the regression error standard deviation $\sigma^{(b)}$. [this step accounts for the parameter uncertainty due to estimation, because given the bootstrapped returns, you estimate the (bootstrapped) parameters as if the bootstrap sample were the true sample.]
Step 6: simulate a future return $R_{T+1}^{(b)} = a^{(b)} + b^{(b)} DY_T + \varepsilon_{T+1}$, where ε_{T+1} is drawn from a normal with zero mean and standard deviation $\sigma^{(b)}$.
Step 7: Store this simulated return.
Step 8: repeat steps 3-7 many times, say for *b*=1,...,1000.
Step 9: Use the simulated returns from step 7 (so all 1000 for example) to do mean variance optimization.

g) Explain 3 different methods to test for mutual fund persistence. Review the main findings in the literature here.

h) Carefullly explain the Fama-McBeth procedure for testing the CAPM (or APT).

TRIAL QUESTION 4.

Consider the standard Fama-French model,

(1) $R_{it} - r_t = \alpha_i + \beta_i (R_{mt} - r_t) + \gamma_i SMB_t + \delta_i HML_t + e_{it}$.

a) Explain how to construct SMB_t using quartile-based portfolios. (be clear!)

b) Assume the error terms e_{it} are uncorrelated (both time-series wise and cross-sectionally. You have a universe of 4000 stocks with the following properties.

Stock type	number of stocks of this type	α_i	β_i	γ_i	δ_i
A	1000	0	1.0	0.0	0.0
B	1000	0	0.5	1.0	0.0
C	1000	0	1.5	-0.5	-0.5
D	1000	0	1.0	0.0	-1.0

Further, all idiosyncratic variances are equal to $\sigma_i^2 = 0.01$. The factors satisfy

$$\begin{pmatrix} R_{mt} - r_t \\ SMB_t \\ HML_t \end{pmatrix} \sim \mathrm{N}(\mu_F, V_F) = \mathrm{N}\left(\begin{pmatrix} 4\% \\ 2\% \\ 3\% \end{pmatrix}, \begin{pmatrix} 4\% & 1\% & 1\% \\ 1\% & 2\% & -0.5\% \\ 1\% & -0.5\% & 1.5\% \end{pmatrix} \right)$$

Assume you hold a portfolio of 1M with *all* the A-type stocks. What is the percentage of this portfolio's return variance that is systematic? (2 digits)

c) What if you only held 10 A-type stocks in your 1M portfolio (and no other stocks). How would your answer to (b) change?

d) Also compute the VaR of the portfolio in (c) and its systematic VaR. (2 digits) Compare the percentage contribution of the systematic VaR to the total VaR.

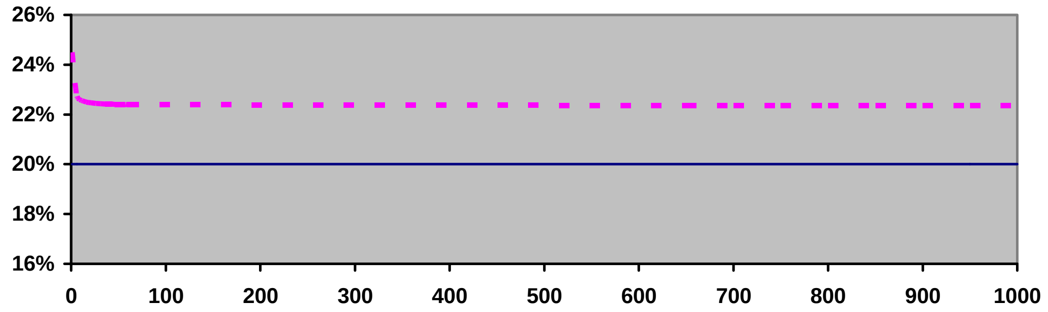
e) Explain why the answers to (c) and (d) are or are not different.

f) Assume that instead as mentioned in the above table, $\alpha_C = 0.02 \neq 0$. What does this mean/imply? Explain some possible causes for $\alpha_C > 0$.

g) Construct a zero-cost portfolio of stocks to exploit the positive alpha in (f) without incurring systematic risk exposure. (Assume you can go long and short in stocks). What are the risk (standard deviation) and expected payoff of this portfolio?

h) What is the number of available C-type stocks were 10 instead of 1000. What would change to your answer in (g)?

i) Assume you are given model (1) above and the table in part (b). Consider you build a portfolio of A-type stocks only. **But** when you make a graph of the portfolio standard deviation against the number of stocks included in your portfolio, the graph looks something like this. (dashed pink curve)



First explain the graph. Second, discuss what the graph tells about the entries in line A of the table in part (b) or about model (1) above.

TRIAL QUESTION 5.

Assume there are only 10 possible realizations of the stock return, namely -10%, -7%, -4%, -1%, 2%, 5%, 8%, 11%, 14%, 17%. All outcomes have equal probability.
Also assume you have 10 digital options (are Arrow-Debreu securities). Each of these options pays out an amount of $1 in one particular state of the world only. So one option pays $1 if and only if the realized return is -10%, another option pays $1 if and only if the realized return is -7%, and so on. Denote the prices of the options by p_i .

a) State the fundamental asset pricing equation using a stochastic discount factor (SDF). From that, derive an expression for the SDF in terms of the option prices only.

b) Assume you have a utility function of the following type

$$(1) \qquad U(W) = \begin{cases} W & \text{if } W \geq A, \\ 3W - 2A & \text{if } W < A. \end{cases}$$

Let $A=W_0=100$. Sketch the definition of loss aversion and show how this utility function satisfies this definition.

c[difficult]) Assume you have the following price table.

Stock Return	-10%	-7%	-4%	-1%	2%	5%	8%	11%	14%	17%
Probability	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Price	1.11	1.08	1.04	1.01	0.98	0.95	0.93	0.90	0.88	0.85

Show why the optimal portfolio if you are an expected utility optimizer with utility as given in (1), has a short position in the -10%-option, and positions of 100 in all the others.
[tip: if you had 200 options for state -1%, and zero for all the others, what is an easy way to improve your utility the most?]

d[difficult]) What would change if you had $A=10$ and still $W_0=100$?

e) What is the riskfree rate implied by the table in part (c)?

f) How would you implement your solution to (c) and (d) if only plain vanilla options were available in the market.

g) What would you do if you can do dynamic rebalancing of your asset portfolio, but you can only invest in stocks and a riskfree asset each time (so not in the digital or plain vanilla options)?

h) Consider a call option entitling you to buy 100 stocks at the current stock price at the same moment the digital options expire. Approximate the price of this option given the prices of the digital options in the table above. Provide the details of how you set up your computations.

i) Discuss the behavioral biases of loss aversion, ambiguity aversion, and over-confidence.

j) What are the differences between expected utility theory and prospect theory. Discuss when and why you should use either of these theories.

TRIAL QUESTION 6.

Consider the following simple world, where prices of a risky asset are given by

t=0		t=1		t=2	
100	up	116	up-up	119	
			up-down	115	
	down	90	down-up	115	
			down-down	85	

All ups and downs happen with equal probability of 50%. The one-period riskfree rate is 1% at all times. Your utility function is given by

(1) $U(W) = -0.25W^{-4}$.

a) Compute the one-period stochastic discount factor (SDF) at time 0, at time 1 in the up-state, at time 1 in the down-state.

b) Use the SDF to price an at the money call option and an at the money put option, both maturing at time t=2. Does put-call-parity hold?

c) Show whether -94.04% is the optimal fraction invested in the risky asset at time t=1 if you are in the up-state?

d) Similar for 92.57% in the down-state?

e) Show that the optimal fraction invested in the risky asset at time t=0 if you are a one-period expected utility maximizer is 24.20%? [so a short-horizon investor]

f) Your optimal fraction invested in the risky asset at time t=0 if you are a two-period expected utility maximizer is 36.90%. [so a long-horizon investor]
Explain why this is different from (e).

h) Explain why in a long-horizon investor problem the investor may or may not hold a larger fraction of risky assets if (s)he is younger. (sketch some model assumptions that would be needed to obtain such a result)

Trial Q4

a) Step 1: at time t, sort a sample of stocks with respect to Market capitalization. The sample can be e.g. 4000 US stocks.

Step 2: Take the top 25% of the sorted stocks and put them in a portfolio. Compute the return on this portfolio from t to $t+1$. Call it r_t^B. (assuming the top 25% have the <u>largest</u> market cap.)

Step 3: Similarly for the bottom 25%. Call it r_t^S.

Step 4: Set $SMB_t = r_t^S - r_t^B$.

Step 5: repeat steps 1-4 for every period t in the sample.

b) Variance of the portfolio is

$$\sigma_p^2 = (1 \ 0 \ 0)\, V_F \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{\sigma_i^2}{1000} = 0.04 + 0.00001 = 0.04001$$

So % systematic of total is $\frac{0.04}{0.04001} \times 100 = 99.98\%$

c) $$\sigma_p^2 = (1 \ 0 \ 0)\, V_F \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{\sigma_i^2}{10} = 0.04 + 0.001 = 0.041$$

So % is $\frac{0.04}{0.041} \times 100 = 97.56\%$

d) VaR: $-1M \cdot \left(r + (1 \ 0 \ 0)\mu - 1.645\, \sigma_p\right) = -1M \cdot \left(0.07 - 1.645 \cdot \sqrt{0.041}\right)$

(I assume 95% VaR)

(I assume $r_t = 3\%$)

$= € 263\,087$

VaR_{syst}: $-1M \left(0.07 - 1.645 \sqrt{0.4}\right) = € 259.000$

$\Rightarrow 98.45\%$

e) VaR is quantile/percentile based, so not only the variance enters the VaR, but also the expected returns. Moreover, the VaR uses the standard deviation and not the variance. For zero mean, the squared % proportion for the VaR should equal the ratio of variances.

Trial Q4

f) There is abnormal return in C-type stocks that is not explained by the Fama-French systematic risk factors.
Possible causes include: better information for project/stock selection (depending on context), more info, better info processing skills, omitted systematic/priced risk factors

g) long in C (i.e. all C-stocks, amount € $\frac{1}{1000}$). eliminate market exposure by short 1.5 A (i.e. all A-stocks amount € $-1.5/1000$). eliminate SMB exposure by long $\frac{1}{2}(B - \frac{1}{2}A)$. eliminate HML by short $\frac{1}{2}(D-A)$. Short (lend) € 0.25

Total: $C - 1\frac{1}{2}A + \frac{1}{2}(B - \frac{1}{2}A) - \frac{1}{2}(D - A) =$

$-1\frac{1}{4}A + \frac{1}{2}B + C - \frac{1}{2}D$ + lend € 0.25 ⇒ zero cost

$$\mathbb{E}[P_{any}] = \begin{pmatrix} -1\frac{1}{4} & \frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0.02 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 4\% \\ 2\% \\ 3\% \end{pmatrix} \right]$$

(β, γ, δ) - for portfolio without syst. risk exposure!

$$= 0.02 + \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4\% \\ 2\% \\ 3\% \end{pmatrix} = 0.02$$

$$\sigma_p^2 = 0 + \sigma_i^2 \left(\frac{(-1\frac{1}{4})^2 \cdot 1000}{1000^2} + \frac{(\frac{1}{2})^2 1000}{1000^2} + \frac{1 \cdot 1000}{1000^2} + \frac{(-\frac{1}{2})^2 1000}{1000^2} \right) = 0.0000030625 \rightarrow \sigma_p \approx 0.55\%$$

↑ idiosyncratic risk still there.

Trial Q4

h) (Other weightings also possible)

€ $\frac{-1\frac{1}{4}}{1000}$ in each A-stock € $\frac{1/2}{1000}$ in each B stock — zero cost

€ $\frac{1}{10}$ in each C-stock € $\frac{-1/2}{1000}$ in each D stock; lend € 0.25

Portfolio β : $\frac{-1\frac{1}{4}}{1000} \cdot 1000 \cdot 1 + \frac{1/2}{1000} \cdot 1000 \cdot (0.5) + \frac{1}{10} \cdot 10 \cdot (1.5) + \frac{-1/2}{1000} \cdot 1000 \cdot 1 = 0$

Similarly: portfolio $\gamma, \delta = 0$

Portfolio expected payoff:

$\mathbb{E}[\text{Payoff}] = 0.02$

Portfolio variance:

$$\sigma_p^2 = 0 + \frac{(-1\frac{1}{4})^2 \, 0.01}{1000} + \frac{\frac{1}{4} \cdot 0.01}{1000} + \frac{0.01}{10} + \frac{\frac{1}{4} \cdot 0.01}{1000}$$

$$= 0.001021 \Rightarrow \sigma_p \approx 3.19\%$$

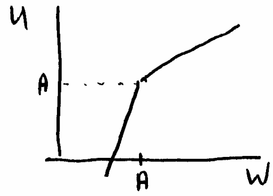
Trial Q5

a) $\mathbb{E}_t[M_{t+1}(1+R_{i,t+1})] = 1$ for all i

This holds for all options, so

$\mathbb{E}_t\left[M_{t+1}\dfrac{A^i_{t+1}}{p_i}\right] = \frac{1}{10}M_{i,t+1}\cdot\frac{1}{p_i} = 1 \iff M_{i,t+1} = 10\,p_i$

b) Loss averse agents have a kink in their utility function around a reference point. They dislike shortfall wrt. this reference point more than they like gains above this ref. point. In this case:



a clear kink around A.

e) $\sum p_i = 0.973 \Rightarrow r^f = 2.77\%$

f) (c),(d) → long far out of the money calls.

if $A=200$ → short far out of the money puts

g) dynamically replicate (f) or the digital options / arrow debreu securities themselves. For (c) this means long stock only if huge increases in stock prices.

h) Price stock: P_0.

Payoff: state i: 0 if $r_i < 0$; $100\cdot P_0\cdot r_i$ if $r_i > 0$

Price: $\sum_i 100\,P_0\cdot r_i\cdot p_i = 100\,P_0\sum_{i=5}^{10} r_i\,p_i = 5.082\,P_0$

Expected payoff: $\frac{100}{10}P_0\sum_{i=5}^{10} r_i = 5.7\,P_0$

Expected return: 12.16%

Trial Q5

c) d) If you have 200 (−12)-options, sell nine, get 0.909, buy one of each of the others @ cost 0.872 (so even money left).

Effect on utility (expected): $\Delta U =$

$$-\frac{9}{10}^{\text{(sells)}} + \frac{3\cdot 9}{10}^{\text{buys}} = +\frac{2}{10}\cdot 9 = +1.8$$

So expected utility can be increased buy selling payoffs above A to fill up payoffs below A. Now there are two settings.

I) You have enough W_0 to buy all stakes up to payoff A. With the money left, you buy the highest return asset, the type (+17%)-option.

II) You have not enough money. Then you short one stake to fill up all the others. Most effective is the most expensive stake, as this will provide you the most funds per unit of shortfall (in that stake) to be used to fill up the other stakes. In this way, you limit the maximum shortfall below A.

Trial Q6

a) @ $t=0$,

(1) $\mathbb{E}[M_1 \cdot P_{t+1}] = \frac{1}{2} M_1^u \, 116 + \frac{1}{2} M_1^d \, 90 = 100$

(2) $\frac{1}{2} M_1^u + \frac{1}{2} M_1^d = \frac{1}{1.01} \Rightarrow \frac{1}{2} M_1^u 90 + \frac{1}{2} M_1^d 90 = \frac{90}{1.01}$

$13 M_1^u = 100 - \frac{90}{1.01} \iff M_1^u \approx 0.8378$

$M_1^d = \frac{2}{1.01} - M_1^u \approx 1.1424$

@ $t=1, u$

(1) $\frac{119}{2} M_2^{uu} + \frac{115}{2} M_2^{ud} = 116$

(2) $\frac{115}{2} M_2^{uu} + \frac{115}{2} M_2^{ud} = \frac{115}{1.01}$

$2M_2^{uu} = 116 - \frac{115}{1.01} \iff M_2^{uu} \approx 1.0693$

$M_2^{ud} = \frac{2}{1.01} - M_2^{uu} \approx 0.9109$

@ $t=2, d$

(1) $\frac{115}{2} M_2^{du} + \frac{85}{2} M_2^{dd} = 90$

(2) $\frac{85}{2} M_2^{du} + \frac{85}{2} M_2^{dd} = \frac{85}{1.01}$

$M_2^{du} \approx 0.3894$

$M_2^{dd} \approx 1.5908$

b) Call: $\mathbb{E}[M_2 (S_2 - 100)^+] =$

$\frac{1}{4}\left(1.0693 \cdot 19 \cdot 0.8378 + 0.9109 \cdot 15 \cdot 0.8378 + 0.3894 \cdot 15 \cdot 1.1424 + 0\right) \approx 8.785$

Put: $\mathbb{E}[M_2 (100 - S_2)^+] = \frac{1}{4} \cdot 1.1424 \cdot 1.5908 \cdot 15 \approx 6.815$

$\Rightarrow$ Put-Call Parity

$\text{Call} + \frac{100}{1.01^2} \overset{?}{=} \text{Put} + 100$

$106.815 \approx 8.785 + \frac{100}{1.01^2} = 106.815$ ✓

c) $\max -\frac{1}{4}\left[\frac{1}{2}\left(1.01 + \alpha\left(\frac{119}{116} - 1.01\right)\right)^{-4} + \frac{1}{2}\left(1.01 + \alpha\left(\frac{115}{116} - 1.01\right)\right)^{-4}\right] \Rightarrow$

$\left(1.01 + \alpha\left(\frac{119}{116} - 1.01\right)\right)^{-5} \cdot \left(\frac{119}{116} - 1.01\right) + \left(1.01 + \alpha\left(\frac{115}{116} - 1.01\right)\right)^{-5}\left(\frac{115}{116} - 1.01\right)$

$\alpha = -0.9404$: $0.016258 - 0.01626 \approx 0$

d, e) similarly

Trial Q6

f) Clearly, there is predictability of expected returns here.
In the up-state, $E_1[R_2] = 0.86\%$.
In the down-state, $E_1[R_2] = 11.11\%$
So the risky asset hedges against changes in the investment opportunity set (or against re-investment risk). So apart from the myopic demand from (e), there is a positive hedging demand for the risky asset.

g) If she has power utility, returns are iid (unpredictable), and if there is no labor income, long and short term investors hold the same portfolio.
If there is predictability, the long-horizon investor has an additional intertemporal hedging demand (pos. or neg.)
Other utility functions may also lead to differences in long- and short term behavior.
Labor income may crowd out fixed income and cause larger fractions of risky assets if human capital is high (i.e. if you are young).