

Answers

1A

Rank stocks in portfolios according to EXPECTED returns. These expected returns are obtained by multiplying firm characteristics like past returns, book to price, size, etc. with coefficients that are obtained from cross-sectional regressions of (past) returns on lagged versions of the same attributes.

[obvious mistake: rank stocks according to past returns. This answer earns no points.]

1B

SML: return spread on portfolio of small caps versus large caps. Large caps: rank (high to low) stocks according to market cap. Take average (cross sectional) return of first decile (10%) of stocks returns.

HML: return spread on value (high B/M) versus growth (low B/M) stocks. Value: rank (high to low) stocks according to B/M. Take average (cross sectional) return of first decile (10%) of stocks returns.

1C

A: skill of manager. Part of expected return not explained by systematic risk exposures.

m: beta or market exposure. Highly significant.

s: size factor exposure.

h: HML factor exposure.

The t-stats indicate that both portfolios have a significant exposure to size. H has a large cap bias. L has a small cap bias (given the sign). Similarly, H has a growth bias, though insignificant at the 5% level. L has a value bias, strongly significant. The regression model fits quite well given the high R^2 values.

1D

$H-L \rightarrow 0.0041 - (-0.0060) = 1\%$ per month, or 12% per year.

1E

We would need long short portfolios also in other (zero alpha) portfolios that have exposures to the same systematic risk factors.

2A

See paper. Key thing: compute residuals on regressions with constant term. Do bootstrap samples of return from regression residuals, but with constant term restricted to zero. Do regression on the bootstrapped return sample including the regression again. Choose the cross sectional max (t-value) of the alpha (=constant term) from the bootstrap return regressions. Look at the distribution of this max statistic over many bootstrap samples.

2B

$$M = a + bR_m$$

$$E[R_i - r] = -\text{cov}(M, R_i)/E[M] = -b \text{cov}(R_m, R_i)/E[M]$$

Check for $R_i = R_m$, we obtain

$$E[R_m - r] = -b \text{cov}(R_m, R_m)/E[M] = -b \text{var}(R_m)/E[M], \text{ solve } b \text{ from this and obtain}$$

$$E[R_i - r] = -b \text{cov}(R_m, R_i)/E[M] = -(-E[R_m - r] E[M] / \text{var}(R_m)) \text{cov}(R_m, R_i)/E[M] \rightarrow$$

$$E[R_i - r] = \text{cov}(R_m, R_i) E[R_m - r] / \text{var}(R_m) = \beta E[R_m - r]$$

2C

$$M = a + bR_m + cZ$$

FAPE holds for r , R_m , and Z . This gives 3 equations in 3 unknowns (a,b,c):

$$1 = E[M(1+r)] = (a + b E[R_m] + c E[Z])(1+r)$$

$$1 = E[M(1+R_m)] = a E[1+R_m] + b E[R_m(1+R_m)] + c E[Z(1+R_m)]$$

$$1 = E[M(1+Z)] = a E[1+Z] + b E[R_m(1+Z)] + c E[Z(1+Z)]$$

2D

Given the SDF M and a return R , compute the covariance $\text{cov}(M, R)$ and the expectation $E[M]$. If $E[R] > -\text{cov}(M, R)/E[M]$, there is abnormal return.

2E

Given time series observations of M_t and R_t , compute the t-statistic of the (time series) mean of $R_t - \text{cov}(R_t, M_t)/E[M_t]$. If it is greater than its positive critical value, the abnormal return is positive.

2F

$M_{t,t+1} = M_{t+1} * \dots * M_{t+1}$. If discount factors are non-stochastic, we obtain

$P_t = E[P_{t+1} * M_{t,t+1}] = M_{t+1} * \dots * M_{t+1} * E[P_{t+1}]$ as in any expected discounted cash flow valuation model. Easiest to see if $M_t = 1/(1+k)$ for any time t . In that case $P_t = E[P_{t+1} * M_{t,t+1}] = E[P_{t+1}]/(1+k)$.

3A

$$1 = E_t[M_{t+1} (1+R_{t+1})]$$

$$E_t[R_{t+1}-r_t] = -\text{cov}_t[M_{t+1}, R_{t+1}] / E_t[M_{t+1}]$$

3B

$$\text{ARA: } -U''/U' = \gamma / (C - d)$$

$$\text{RRA: } -C U''/U' = \gamma C / (C - d)$$

3C

Utility of future consumption depends on present consumption levels. This is habit formation. So large consumption today means that consumption utility tomorrow will be benchmarked against 80% of today's level.

3D

$$\max E \left[\frac{C_0^{1-\gamma}}{1-\gamma} + \beta \frac{((W_0 - C_0)\alpha'(1+R) - kC_0)^{1-\gamma}}{1-\gamma} \right] + \lambda(1-t'\alpha)$$

FOC C_0 :

$$(1) \quad E \left[C_0^{-\gamma} + \beta ((W_0 - C_0)\alpha'(1+R) - kC_0)^{-\gamma} (-\alpha'(1+R) - k) \right] = 0$$

FOC α :

$$(2) \quad E \left[\beta ((W_0 - C_0)\alpha'(1+R) - kC_0)^{-\gamma} (W_0 - C_0)(1+R) \right] = \lambda t$$

FOC λ :

$$(3) \quad 1 = t'\alpha$$

Multiplying (2) by α' and using (3) and (1), we get

$$(4) \quad \lambda = E \left[\beta ((W_0 - C_0)\alpha'(1+R) - kC_0)^{-\gamma} (W_0 - C_0)\alpha'(1+R) \right] = E \left[(W_0 - C_0) (C_0^{-\gamma} - \beta k ((W_0 - C_0)\alpha'(1+R) - kC_0)^{-\gamma}) \right]$$

Dividing both sides of (2) by the lambda from (4), we get the FAPE

$$E[M(1+R)] = t$$

with

$$M = \frac{\beta ((W_0 - C_0)\alpha'(1+R) - kC_0)^{-\gamma}}{E \left[C_0^{-\gamma} - \beta k ((W_0 - C_0)\alpha'(1+R) - kC_0)^{-\gamma} \right]},$$

which for $k = 0$ collapses to the familiar expression.

3E

$k > 0$ implies greater risk aversion of the representative agent, therefore less risky investments, a higher equilibrium risk premium and a lower equilibrium riskfree rate for the same gamma value as under $k = 0$. Consequently, gamma may be lowered for $k > 0$ to match empirical (historical) risk premia, thus partially solving the EPP.

4A

See book.

4B

See book.

4C

Correlations between assets: positive correlation implies less diversification.

Correlations with the SDF (or marginal utility): positive correlation implies more consumption smoothing and therefore will require lower premia.

Correlations with future expected returns: positive correlation of realized returns with future expected returns gives less smoothing for long term investors, so less desirable.

4D

A regression of this type will have a non-zero slope coefficient if $\text{cov}(r_t, r_{t-1})$ is large.

Note that $\mu_t = r + \lambda \sigma^2$. As variances are time varying, the cov of the returns will relate directly to the cov of the time varying variances. This is strong given the model, such that we will find a positive coefficient on past returns in a regression. If volatilities shoot up, they will remain high for some time. But also, if volatilities shoot up, expected returns shoot up, and returns will on average lie at a somewhat higher level for some time. This causes the positive regression coefficient.

4E

EMH states that no abnormal returns are possible beyond the equilibrium returns. As equilibrium returns are time varying here in a predictable way, predictability does not violate the EMH for this model.