Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration

Programs:	M.Sc. Finance, M.Sc. Quantitative Finance
Exam:	Investments 4.1
Course code:	60412040
Date:	October 27, 2006
Time:	8:45 – 11:30
Duration:	2 hours, 45 minutes
Parts:	The exam has 4 questions, each with 5 subquestions.
Grading:	Each of the four questions in the exam yields you a maximum of 10 points. All subquestions are **equally weighted**. Your total score cannot exceed 40 points. The grade for this written exam is obtained by dividing the points scored by 4. Perhaps redundantly: the written exam makes up 70% of your final grade. The remaining 30% is scored by the cases. The exam can be re-taken. The cases cannot.
Results:	Results will be made known as soon as possible, but at the latest Monday, November 13, 2006.
Inspection:	You can inspect your marked exam papers Tuesday, November 14, 9:00am. The room will be announced via the monitor system.
Remark:	Provide complete answers (including computations where appropriate). Always provide motivation/explanation of your answer, even if this is not mentioned explicitly in the question. A short ‘yes’ or ‘no’ will never do as an answer. Use your time efficiently.

Good luck!

This document has 5 pages.

If $X \sim N(m,s^2)$, then $E[\exp(X)] = E[e^X] = \exp(m + 0.5s^2)$.

If $X \sim N(\mu, V)$, then $w' X \sim N(w'\mu, w'Vw)$.

$X = \exp(\ln(X))$; $\exp(X) \exp(Y) = \exp(X+Y)$; $\ln(X^a) = a \ln(X)$

$ax^2 + bx + c = 0 \Leftrightarrow x = (-b \pm (b^2 - 4ac)^{1/2})/(2a)$

QUESTION 1

Part a.
Assume you have a utility function $U(W) = -\exp(-3W)$. Compute the coefficient of absolute risk aversion and the coefficient of relative risk aversion.

Part b.
Let R denote the vector of risky returns. Let the riskfree rate be 3% and set your initial wealth to unity, $W_0=1$. There are n risky assets. Assume in addition to part (a) that the risky returns R are normally distributed with mean vector μ and covariance matrix V, $R \sim N(\mu, V)$. Derive the formula for the optimal asset allocation/asset demand for the risky assets if you are an expected utility optimizer.
[You may assume that there is a riskfree asset available as well.]

Part c.
Show how the first order condition (FOC) of an expected utility optimizer with general utility function $U(W)$ gives rise to the fundamental asset pricing theorem. (Derive the theorem from the FOC, state it in 2 forms: one using expected individual excess returns, one using (stochastically) discounted gross returns,).

Part d.
Assume returns behave as in part (b). Also assume that all investors in the economy have a utility function of the form $U(W) = -\exp(-b \cdot W)$, where the parameter $b > 0$ may be different for each investor. Show that two-fund separation holds.
[tip: what is the relation between the risk aversion coefficient and the composition of the risky part of the investment portfolio?]

Part e.
From parts (a-d) it follows that the stochastic discount factor (SDF) in this setting is linear in the 'market' portfolio return. Assume asset XYZ's return has a covariance of 0.04 with the market return. The market return has an expectation of 7% and a standard deviation of 20%. The riskfree rate is still 3%. What then is the required expected excess return on asset XYZ?

Question 2.

Part a.
How would you construct the Fama-French size factor yourself given time series of 1500 European stocks?

Part b.
You have built 3 portfolios, P1, P2, P3. You perform Fama-French regressions with the portfolio returns and obtain Table 1.

Table 1

This table contains the regression results of portfolio returns (P1, P2, P3) on Fama-French factors: excess market return R_m-r, size SMB, and value HML. Standard errors are in parentheses. s is the standard deviation of the regression error. Bold entries in the table indicate the coefficient is significant at the 5% level.

	P1 – r	P2 – r	P3 – r
constant	0.00	0.00	**0.51**
	(0.10)	(0.15)	**(0.08)**
R_m– r	**1.13**	**1.01**	**0.98**
	(0.10)	**(0.25)**	**(0.15)**
SMB	**-1.05**	**0.89**	**0.55**
	(0.33)	**(0.12)**	**(0.11)**
HML	0.00	0.00	0.00
	(0.21)	(0.33)	(0.25)
R^2	89%	90%	78%
s (regression error standard deviation)	7.6%	7.2%	9.6%

Abstract from parameter uncertainty.
You have estimated the mean and covariance matrix of the systematic risk factors to be

$$\begin{pmatrix} R_m - r \\ SMB \\ HML \end{pmatrix} \sim \left(\begin{pmatrix} 7.6\% \\ -1.3\% \\ 6.6\% \end{pmatrix}, \begin{pmatrix} 2.4\% & 0.3\% & -1.1\% \\ 0.3\% & 2.1\% & -0.9\% \\ -1.1\% & -0.9\% & 1.7\% \end{pmatrix} \right).$$

Explain at least two possible sources for the significantly positive constant term in the regression involving P3.

Part c.
Using the information of part (b), compute the total risk of portfolio P3. Which part of this is systematic risk?

Part d.
Assuming you can go long and short in all three portfolios, build a hedge fund that exploits the alpha of portfolio P3 without exposing yourself to any of the systematic risk factors. Give the risk of this hedge fund portfolio return.

Part e.
Describe conditions under which exploiting this alpha of P3 would be a sensible investment strategy, and conditions under which it would be a flawed investment strategy.

Question 3.

Part a.
Consider the Consumption CAPM (C-CAPM) with constant relative risk aversion (CRRA) utility function. The stochastic discount factor (SDF) is given by

$$M_{t+1} = 0.95\exp(-\gamma \Delta c_{t+1}), \quad \Delta c_{t+1} \sim \mathrm{N}\left(2\%, 0.1\%\right).$$

If the riskfree rate is 3%, what is the value of the risk aversion coefficient γ such that the fundamental asset pricing theorem is satisfied for the riskfree rate?

Part b.
What value of γ would be needed for the fundamental asset pricing theorem to hold for the market return if the market return $R_{m,t+1}$ $(r_{m,t+1} = \ln(1+\mathrm{R}_{m,t+1}))$ satisfies

$$\begin{pmatrix} \Delta c_{t+1} \\ r_{m,t+1} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 2\% \\ 4\% \end{pmatrix}, \begin{pmatrix} 0.10\% & 0.15\% \\ 0.15\% & 2.5\% \end{pmatrix}\right).$$

Part c.
(Briefly) describe at least four modifications of the above C-CAPM with CRRA utility that help in solving the equity premium puzzle.

Part d.
You are visited by a venture capital (VC) fund manager. Define the returns of this VC manager as $R_{VC,t+1}$. Using observed consumption growth, the track record of the VC manager, and your best estimate of the coefficient γ, you find an expected excess return on the VC fund of 4%, a riskfree rate of 3%, and a covariance of the VC fund return with the SDF of 4%. Would you advise to invest in this VC manager's fund? Why or why not?

Part e.
Financial advice often reads "stocks when you're young, bonds when you're old." Briefly describe two theoretical frameworks in which this advice is indeed optimal. Also briefly provide the intuition for why the advise is optimal in these settings.

Question 4.
Part a.
Define the VaR at the 95% confidence level. (Briefly) discuss (not only mention) two advantages and two disadvantages of VaR as a risk measure compared to the return standard deviation.

Part b.
Describe two ways to estimate the 1-month 95%-VaR of a set of Mutual Fund monthly returns.

Part c.
You estimated the 2-month 95%-VaR using 2-month overlapping data and one of your two estimation methods from part (b). Describe a bootstrap procedure to compute the standard deviation of your 2-month VaR estimate based on overlapping data.

Part d.
Describe (at least) two anomalies found in the empirical finance literature. With each anomaly, also highlight why it is an 'anomaly'.

Part e.
Assume the riskfree rate is 3% and the market expected excess return is 4% with standard deviation 20%. Explain under what conditions you (i) would and (ii) would not want to hold a security with an expected return of 2.5% and return standard deviation of 30%.