ANSWERS

Question 1.
Part a.
$U'(W) = 3\exp(-3W);\ U''(W) = -9\exp(-3W);\ R_A = -U''/U' = 3;\ R_R = W \cdot R_A = 3W.$
Part b.

$$\begin{aligned}&\max\ \mathrm{E}\left[-\exp(-3(1.03+\alpha'(R-0.03))\right] =\\&\max\ -e^{3.09}\mathrm{E}\left[\exp(-3\alpha'(R-0.03))\right] =\\&\max\ -e^{3.09}\mathrm{E}\left[\exp(\mathrm{N}\left(-3\alpha'(\mu-0.03),9\alpha'V\alpha\right))\right] =\\&\max\ -e^{3.09-3\alpha'(\mu-0.03)+0.5\cdot 9\alpha'V\alpha}\end{aligned}$$

$$\begin{aligned}&\mathrm{FOC}:\\&-e^{3.09-3\alpha'(\mu-0.03)+0.5\cdot 9\alpha'V\alpha}\cdot\left(-3(\mu-0.03)+9V\alpha\right)=0 \Leftrightarrow\\&\left(-3(\mu-0.03)+9V\alpha\right)=0 \Leftrightarrow\\&9V\alpha = 3(\mu-0.03) \Leftrightarrow\\&\alpha = \tfrac{1}{3}V^{-1}(\mu-0.03)\end{aligned}$$

Part c.

$$\begin{aligned}&\mathrm{FOC}:\ \mathrm{E}\left[U'(W)(R-0.03)\right]=0\\&\text{Relabel } M = U'(W), \text{ then}\\&0=\mathrm{E}\left[M(R-0.03)\right]=\mathrm{cov}(M,1+R)+\mathrm{E}\left[M\right]\mathrm{E}\left[R-0.03\right] \Leftrightarrow \mathrm{E}\left[R-0.03\right] = -\mathrm{cov}(M,1+R)/\mathrm{E}\left[M\right].\end{aligned}$$

$$\begin{aligned}&\mathrm{E}\left[R-0.03\right]=\mathrm{E}\left[1+R-1.03\right]=\mathrm{E}\left[1+R\right]-\left(\mathrm{E}\left[M\right]\right)^{-1}=-\mathrm{cov}(M,1+R)/\mathrm{E}\left[M\right] \Leftrightarrow\\&\mathrm{E}\left[1+R\right]\mathrm{E}\left[M\right]-1=-\mathrm{cov}(M,1+R) \Leftrightarrow \mathrm{E}\left[(1+R)M\right]=1.\end{aligned}$$

Part d.
From part (b), it follows that for general risk aversion coefficient $b > 0$ we get

$$\alpha = b^{-1}V^{-1}(\mu - 0.03)\ ,$$

which is in all cases a multiple of the *same* portfolio weights $V^{-1}(\mu - 0.03)$. So all investors hold a combination of the riskfree asset and this *same* risky portfolio, the weights between this riskfree and *same* risky portfolio depending on the value of their risk aversion parameter.

Part e.
The question implies that CAPM holds. As a result, we compute the beta, in this case $0.04/0.20^2 = 1$. So the required expected excess return is 1*(7%-3%) = 4%.

QUESTION 2.

Part a.

At time *t*, sort all companies with respect to market cap. Make portfolios, e.g., deciles. Compute the portfolio return for the small market cap decile, and subtract the portfolio return for the large market cap decile. The resulting number is your time *t* observation of SMB. Move on to the next time.

Part b.

Positive alpha may be due to better information (e.g., timelier) or better information processing skills (e.g., better models for forecasting, better exploitation of correlation with readily observable variables, ...)

Part c.

Systematic/Total = 78% (R^2). Idiosyncratic is 0.096^2. So total risk is $0.096^2/0.22$ = 4.189%.

Alternative:

systematic risk is

$0.98^2*2.4\% + 0.55^2*2.1\% + 2*0.98*0.55*0.3\% = 3.26361\%$

idiosyncratic risk is $0.096^2 = 0.9216\%$

syst/total = 3.26361/4.18521 = 78%

Part d.

Solve

1.13x + 1.01y + 0.98 = 0

-1.05x + 0.89y + 0.55 = 0

x = -0.1533

y = -0.7988

Long P3, short 0.1533 P1, short 0.7988 P2. 0.0479 borrow (or equity).

Risk: $0.1533^2 7.6\%^2 + 0.7988^2 7.2\%^2 + 9.6\%^2$

Part e.

Sensible if the risk factors and estimated loadings are correct (i.e., if the economic model is correct, this is an abnormal source of profits).

If the model is not correct, e.g., because a systematic risk factor is missing, the alpha may just mop up a premium for a missed systematic risk factor. In that case, there is no abnormal return to be obtained. The economic model is just mis-specified.

QUESTION 3.

Part a.

$$1/1.03 = \mathrm{E}[M] = 0.95\exp(-0.02\gamma + 0.5 \cdot 0.001\gamma^2)$$

$$\gamma = \left(0.02 \pm \sqrt{0.0004 - 0.002 \cdot (-0.021734)}\right)/\,0.001 \to 41$$

Part b.

$$0.95^{-1} = \mathrm{E}\left[\exp(-\gamma \Delta c_{t+1} + r_{t+1})\right] = \exp\left(4\% - 2\%\gamma + 0.5\left(2.5\% - 2\gamma 0.15\% + \gamma^2 0.1\%\right)\right)$$

$$-\ln(0.95) = 4\% - 2\%\gamma + 0.5\left(2.5\% - 2\gamma 0.15\% + \gamma^2 0.1\%\right)$$

$$-\ln(0.95) = 5.25 - 2.15\gamma + 0.05\gamma^2$$

$$\gamma = \left(2.15 \pm \sqrt{4.6225 - 4 * 0.05 * (5.25 + \ln(0.95))}\right)/\,0.1$$

Part c.

Other utility functions could help in decreasing the necessary gamma parameter, e.g., seperating time preference from state preferences (Epstein Zin), using habit formation (higher risk aversion if closer to previous habit levels).
Limited stock market participation: less people to spread the risk over, so larger premia needed.
Consumption is not the only variable; maybe other variables need to enter as well (e.g., in a conditional capm context).
Stock returns may be predictable, giving rise to a more complicated pricing equation for multi-period investors.
Data may be flawed, as it does not contain the key catastrophic event all investors are worried about and therefore want to be compensated for (peso problems).
Investors may not be expected utility maximizers, and therefore the whole model is flawed.
Financial markets may not be as efficient as claimed.
The equilibrium assumptions make use of a representative agent (aggregate consumption), whereas individual consumption or wealth should be used. This is much more volatile, thus increasing the parameter for the γ^2 terms above.

Part d.

You find 4% excess return. But according to the FAPE, the excess return should be -4%*1.03 = -4.12%. So this is a great opportunity (if the pricing model is correct). Of course, you would need to double check the validity of the model for this type of new investment. Are there missed out risk factors, e.g., liquidity, etc..

Part e.

1: If there is labor income/human capital, it will crowd out fixed income largely. As there is much human capital if you're young, it will crowd out the fixed income part of your financial portfolio, causing you to have more equities in your financial assets. Reverse when you are old.
2: If stocks are mean reverting (or hedge against re-investment risk), they become more attractive to long horizon investors. The longer your horizon, relatively speaking the more equities you will hold because of this hedging demand.

QUESTION 4.
Part a.
The maximum loss in the 95% best cases.
The minimum loss in the 5% worst cases.
Advantage:
Clear downward focus: this is more in line with the findings from the psychology/behavioral literature
Expressed in currency: intuitively easier to grasp than the return standard deviation.
Disadvantage:
Not subadditive: the risk of a portfolio need not be smaller than the sum of the (VaR) risks of the constituents
It does not weight the losses in the 5% worst cases; we know nothing about them from the VaR.
In combination with the expected return, the VaR can give degenerate solutions to optimal asset management if derivatives can be used.
It is harder to deal with analytically than the variance of returns. This may be cumbersome in theoretical models.

Part b.
1. order the returns from high to low. Pick the empirical 5^{th} percentile of the returns.Take the negative value of this and multiply it by initial wealth.
2. estimate mean and variance of the returns. Compute initial wealth times (1.645*standard dev – mean).

Part c.
Step 1. Sample (with replacement) a series of 1-month returns of the same length as the original sample.
Step 2. Estimate your VaR using the same method as you did before.
Step 3. Retain the VaR for this bootstrap-sample.
Step 4. Repeat 1-3 many times.
Step 5. Use the standard deviation of the retained (sampled) VaR estimates as your estimate of the standard deviation of the VaR.

Part d.
See chapter 18.

Part e.
(i) if the asset has a (sufficiently) positive covariance with the SDF, this can still be an equilibrium return (or even higher). For example, if the asset has negative beta and most of the 30% risk is idiosyncratic and you hold a well diversified portfolio.
(ii) If the risk is largely systematic and there is a negative covariance with the SDF, you would require a positive risk premium. Therefore, you would not hold the asset in that case.