

Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration

Programs:	M.Sc. Finance, M.Sc. Quantitative Finance
Exam:	Investments 4.1
Course code:	60412040
Date:	May 29, 2006
Time:	12:00 – 14:45
Duration:	2 hours, 45 minutes
Parts:	The exam has 4 questions or 19 subquestions plus 1 bonus subquestion.
Grading:	<p>Each of 19 subquestions in the exam will be equally weighted. The bonus will earn a max of 0.5 points, but you will never get more than 10pt for the total exam.</p> <p>Perhaps redundantly: the written exam makes up 70% of your final grade. The remaining 30% is scored by the cases. The cases cannot.</p>
Results:	Results will be made known as soon as possible, but at the latest Friday, June 8, 2007.
Inspection:	You can inspect your marked exam papers Tuesday, June 12, 9:00am. The room will be announced via the monitor system.
Remark:	Provide complete answers (including computations where appropriate). Always provide motivation/explanation of your answer, even if this is not mentioned explicitly in the question. A short 'yes' or 'no' will never do as an answer. But also be concise/crisp in your answer, or it will take you too much time to write it down. Use your time efficiently.

**Scan for the (in your opinion)
easier questions first.
Good luck!**

This document has 5 pages (this page included)

1a.

Consider the following utility function,

$$U(C) = \frac{\exp(-a \cdot C)}{-a}.$$

What is the coefficient of absolute risk aversion and the coefficient of relative risk aversion.

1b.

Consider a general utility function $U(C)$ and assume you want to maximize

$$\max U(C_0) + \beta \cdot E[U(C_1)],$$

with respect to consumption now and the asset allocation. You can allocate your money to the riskfree asset r^f or to n risky assets. The risky assets have a (n -dimensional) return vector R and an excess return vector $R^e = R - r^f$. The allocations to the risky assets are put into an n -dimensional vector α . At time 1, you consume all your wealth, implying

$$C_1 = (W_0 - C_0) \cdot (1 + r^f + \alpha' R^e).$$

Note there is no restriction on α .

Use the first order condition of the maximization problem with respect to α to show that the expectation of excess returns weighted by marginal utility at time 1 equals zero.

1c.

Rewrite the expression $E[U'(C_1) R^e] = 0$ with $U'(C_1)$ the marginal utility at time 1, into an expression for expected excess returns (or costs of capital) for the risky assets. Also discuss how your result relates to the fundamental asset pricing theorem.

1d.

Explain in words why expected excess returns in an equilibrium economy are higher of the excess returns covary negatively with marginal utility at time 1.

1e.

Assume consumption at time 1 is normally distributed (not[!!] log-normally) with mean 2% and standard deviation 0.5%. Assume that a specific asset i is log-normally distributed, such

that $r_i = \ln(1 + R_i) \sim N(7\%, 4\%)$, with 4% being the variance of the log return. Finally, the covariance between the log-return r_i and consumption at time 1 is 0.05%. Is this asset earning an abnormal return? Why or why not?

2a.

Describe the steps in the Fama-MacBeth procedure for testing the CAPM.

2b.

Consider a vector of excess returns over the riskfree asset R^e . The riskfree return is r^f . The excess returns are normally distributed with mean m and covariance matrix V . Consider yourself a mean-variance optimizer,

$$\max \alpha' m - 0.5 \lambda \alpha' V \alpha .$$

where α is the allocation to the risky assets.

Using the first order condition with respect to α , derive the optimal asset allocation of this mean variance investor.

2c.

Using your formula derived under 2b, show that if two investors have a different value for the risk aversion parameter λ , then (i) the risky part of their portfolio will be different, but (ii) the composition of the risky part of the portfolio (i.e. the investment in risky asset i as a fraction of the investment in the risky part of the portfolio) will be the same.

2d.

Assume the stochastic discount factor takes the form

$$M = a + b \cdot (R_m - r^f) + c \cdot SMB + d \cdot HML .$$

Show using the fundamental asset pricing theorem that this gives rise to the Fama-French model for determining the required return on an asset.

2e.

Assume you performed a market model regression of the portfolio return R_{pt}

$$(R_{pt} - r^f) = \alpha + \beta(R_{mt} - r^f) + e_{it},$$

using OLS. You obtained an alpha of 0.03 over your sample period, with a significant t-value of 2.9. However, (i) for your regression you used monthly observations of quarterly returns (so overlapping observations), and (ii) to compute your t-value, you used normal OLS standard errors rather than Newey-West corrected standard errors.

Give all the steps of a bootstrap procedure to convince me that your α is indeed significantly different from zero.

3a.

Give a brief description in words of the equity premium puzzle and give its relation to the riskfree rate puzzle.

3b.

Give two possible criticisms on the strong results concerning outperformance as presented in the paper of Haugen and Baker.

[Assume Haugen and Baker reported everything truthfully and made no mistakes in their data or programming.]

3c.

We know dividend yields (D/P) are highly persistent over time. Many researchers have found that the market return can be described by the following regression model,

$$R_{mt} = a + b(D/P)_{t-1} + e_t.$$

Explain in words how it can be possible that returns are predictable, but that this predictability cannot be exploited to realize abnormal returns.

3d.

Describe a setting where such predictability can be exploited.

3e. (bonus)

Assume the correct discount factor is M , but you run the regression of excess returns of your portfolio on the excess returns of the stock index and a constant term. You obtain a significant alpha (constant term). Show using the OLS expressions for your estimators in your regression that this alpha may be spurious if the regression errors of your portfolio on M and a constant are correlated with those of the market on M and a constant.

4a.

There has been a long debate on whether the young should invest more in stocks than the old. One explanation has been that young people “play the game” of annually investing in stock more times than do old people, and therefore diversify their risk over time. Provide to competing explanations why young people should invest more in stocks than old people.

4b.

Assume you have estimated the factor sensitivities of a number of excess returns using the regressions

$$R_{it} - r^f = \alpha + \beta_{i1}f_{1t} + \dots + \beta_{iK}f_{Kt} + e_{it} = \beta'_i f_t + e_{it},$$

where the f_t is the vector with factors (e.g., the Fama-French-Carhart factors) and β_i the vector of factor sensitivities for asset i . Some of your assets have significant alphas. You decide to make a portfolio with weight w_{it} in asset i at time t to take *maximum* advantage of the alphas *without* incurring any systematic risk. You are not allowed to take short position. Give the formulas for this optimization problem, i.e., the objective and the constraints.

4c.

You have the 2 factor model

$$R_{it} - r^f = \alpha + \beta_{i1}f_{1t} + \beta_{i2}f_{2t} + e_{it} = \alpha + \beta'_i f_t + e_{it}.$$

Factor 1 has mean 2% and standard deviation 8%, factor 2 has mean 4% and standard deviation 15%. The following numbers hold.

Security type	β_{i1}	β_{i2}	stdev(e_{it})
1	0.25	-0.50	12%
2	-0.50	0.50	15%
3	1.00	1.00	18%

There are 300 securities of each type. Show that an equally weighted portfolio of 160 type 1 and 120 type 2 stocks and 20 type 3 stocks has a zero systematic risk exposure.

4d.

What is the Sharpe ratio of this portfolio under (4c).

4e.

Give another portfolio that also has a zero systematic risk exposure.

How can you make a choice between this new portfolio, and that of (4c) (or any other zero beta portfolio)?