Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration

Programs:	M.Sc. Finance, M.Sc. Quantitative Finance
Exam:	Investments 4.1
Course code:	60412040
Date:	May 29, 2006
Time:	12:00 – 14:45
Duration:	2 hours, 45 minutes
Parts:	The exam has 4 questions or 19 subquestions plus 1 bonus subquestion.
Grading:	Each of 19 subquestions in the exam will be equally weighted. The bonus will earn a max of 0.5 points, but you will never get more than 10pt for the total exam. Perhaps redundantly: the written exam makes up 70% of your final grade. The remaining 30% is scored by the cases. The cases cannot.
Results:	Results will be made known as soon as possible, but at the latest Friday, June 8, 2007.
Inspection:	You can inspect your marked exam papers Tuesday, June 12, 9:00am. The room will be announced via the monitor system.
Remark:	Provide complete answers (including computations where appropriate). Always provide motivation/explanation of your answer, even if this is not mentioned explicitly in the question. A short ‘yes’ or ‘no’ will never do as an answer. But also be concise/crisp in your answer, or it will take you too much time to write it down. Use your time efficiently.

Scan for the (in your opinion) easier questions first.
Good luck!

This document has 5 pages (this page included)

1a.
Consider the following utility function,

$$U(C) = \frac{\exp(-a \cdot C)}{-a}.$$

What is the coefficient of absolute risk aversion and the coefficient of relative risk aversion.

$R_A = a$
$R_R = aC$

1b.
Consider a general utility function $U(C)$ and assume you want to maximize

$$\max\ U(C_0) + \vartheta \cdot \mathrm{E}[U(C_1)],$$

with respect to consumption now and the asset allocation. You can allocate your money to the riskfree asset r^f or to n risky assets. The risky assets have a (n-dimensional) return vector R and an excess return vector R^e=R-r^f. The allocations to the risky assets are put into an n-dimensional vector α. At time 1, you consume all your wealth, implying

$$c_1 = (w_0 - c_0) \cdot (1 + r^f + \alpha' R^e).$$

Note there is no restriction on α.
Use the first order condition of the maximization problem with respect to α to show that the expectation of excess returns weighted by marginal utility at time 1 equals zero.

FOC wrt α :

$$\mathrm{E}\left[u'(c_1)\frac{\partial c_1}{\partial \alpha}\right] = \mathrm{E}\left[u'(c_1) \cdot (w_0 - c_0) \cdot R^e\right] = 0 \Leftrightarrow \mathrm{E}\left[u'(c_1) \cdot R^e\right] = 0$$

1c.
Rewrite the expression $\mathrm{E}[u'(c_1)R^e] = 0$ with $U'(C_1)$ the marginal utility at time 1, into an expression for expected excess returns (or costs of capital) for the risky assets. Also discuss how your result relates to the fundamental asset pricing theorem.

$$\mathrm{E}[U'(C_1) \cdot R^e] = 0 = \mathrm{cov}[U'(C_1), R^e] + \mathrm{E}[U'(C_1)]\mathrm{E}[R^e] \Leftrightarrow$$

$$\mathrm{E}[R^e] = \frac{-\mathrm{cov}[U'(C_1), R^e]}{\mathrm{E}[U'(C_1)]} = \frac{-\mathrm{cov}[U'(C_1)/U'(C_0), R^e]}{\mathrm{E}[U'(C_1)/U'(C_0)]} = \frac{-\mathrm{cov}(M, R^e)}{\mathrm{E}(M)}$$

1d.
Explain in words why expected excess returns in an equilibrium economy are higher of the excess returns covary negatively with marginal utility at time 1.

These returns insure/hedge against consumption risk.
If returns are high when consumption is low (marginal utility is high) these assets are desirable for consumption smoothing. In equilibrium there demand is stronger, driving up the price and therefore driving down the expected returns.

1e.
Assume consumption at time 1 is normally distributed (not[!!] log-normally) with mean 2% and standard deviation 0.5%. Assume that a specific asset i is log-normally distributed, such

that $r_i = \ln(1+R_i) \sim N(7\%,4\%)$, with 4% being the variance of the log return. Finally, the covariance between the log-return r_i and consumption at time 1 is 0.05%. Is this asset earning an abnormal return? Why or why not?

$$E[M(1+R)] = E[\exp(-aC+r)] = E[\exp(N(-0.02a+0.07, 0.005a^2+0.04-0.001a))] = \exp(-0.02a+0.07+0.5\cdot(0.005a^2+0.04-0.001a))$$

Depending on whether this is > 1 (abnormal extra return) or < 1 (abnormal negative return) there is a abnormal return, because the asset yields more (less) than needed to compensate its covariation with marginal utility.

2a.
Describe the steps in the Fama-MacBeth procedure for testing the CAPM.

see book (especially describe 2 passes + computation of average risk premium)

2b.
Consider a vector of excess returns over the riskfree asset R^e. The riskfree return is r^f. The excess returns are normally distributed with mean m and covariance matrix V. Consider yourself a mean-variance optimizer,

$$\max \alpha' m - 0.5\lambda\alpha' V\alpha .$$

where α is the allocation to the risky assets.
Using the first order condition with respect to α, derive the optimal asset allocation of this mean variance investor.

FOC: $m - \lambda V\alpha = 0 \Leftrightarrow \alpha = V^{-1}\alpha / \lambda$

2c.
Using your formula derived under 2b, show that if two investors have a different value for the risk aversion parameter λ, then (i) the risky part of their portfolio will be different, but (ii) the composition of the risky part of the portfolio (i.e. the investment in risky asset *i* as a fraction of the investment in the risky part of the portfolio) will be the same.

If λ increases, the α decreases, so less in the risky assets. As λ is present in every element of the vector α as the same scalar, however, the ratio of α_i to α_j remains constant and independent of lambda, so the composition of the risky portfolio remains the same.

2d.
Assume the stochastic discount factor takes the form

$$M = a + b\cdot(R_m - r^f) + c\cdot SMB + d\cdot HML .$$

Show using the fundamental asset pricing theorem that this gives rise to the Fama-French model for determining the required return on an asset.

The FAPE gives rise to $E[S-r^f] = -\text{cov}(M,S)/E[M]$, and similar for B, H, L, etc. So
$E[SMB] = E[S-r^f] - E[B-r^f] = -(\text{cov}(M,S) - \text{cov}(M,B))/E[M] = -\text{cov}(M,SMB)/E[M]$.
Write this as
$E[RMRF] = -(b\,\text{cov}(RMRF,RMRF) + c\,\text{cov}(RMRF,SMB) + d\,\text{cov}(RMRF,HML))/E[M]$.

E[SMB] = –(b cov(SMB,RMRF) + c cov(SMB,SMB) + d cov(SMB,HML))/E[M].
E[HML] = –(b cov(HML,RMRF) + c cov(HML,SMB) + d cov(HML,HML))/E[M].
So if F'=(RMRF,SMB,HML), m=E[F], and V=Cov(F), w'=(b,c,d), then this is the same as
E[F] = –cov(F,F)w/E[M] = –V*w/E[M].
$E[R_i-r^f] = -cov(R_i,M)/E[M] = -cov(R_i,F)*w/E[M] = -cov(R_i,F)\ V^{-1}\ V\ w\ /\ E[M] =$
$cov(R_i,F)\ V^{-1}\ m = \beta_i^{RMRF}\ m^{RMRF} + \beta_i^{SMB}\ m^{SMB} + \beta_i^{HML}\ m^{HML}$

2e.
Assume you performed a market model regression of the portfolio return R_p,

$$\left(R_{pt} - r^f\right) = \alpha + \beta\left(R_{mt} - r^f\right) + e_{it}\ ,$$

using OLS. You obtained an alpha of 0.03 over your sample period, with a significant t-value of 2.9. However, (i) for your regression you used monthly observations of quarterly returns (so overlapping observations), and (ii) to compute your t-value, you used normal OLS standard errors rather than Newey-West corrected standard errors.
Give all the steps of a bootstrap procedure to convince me that your α is indeed significantly different from zero.

1. For monthly returns, estimate the model. (parameter estimates a and b)
2. sample from the residuals with replacement
3. using the sampled residuals, construct sampled returns bRMRF(t) + e_hat(t), so with a=0.
4. from these sampled monthly returns, construct monthly observations of quarterly returns.
5. using the monthly observations of quarterly returns, estimate the regression model as stated with intercept.
6. Store the t value of the intercept, and repeat 2-5 many times.
7. Estimate the p-value as the fraction of stored t values that exceeds the empirically estimated t value.

3a.
Give a brief description in words of the equity premium puzzle and give its relation to the riskfree rate puzzle.

See book: given standard preferences, unrealistic risk aversion must be imposed to match equilibrium risk premia to observed/realized premia from the past. Riskfree rate puzzle: even if one would accept these unrealistically large risk aversion coefficients, then another puzzle emerges: the risk free rate should have been much higher and more volatile than actually observed.

3b.
Give two possible criticisms on the strong results concerning outperformance as presented in the paper of Haugen and Baker.
[Assume Haugen and Baker reported everything truthfully and made no mistakes in their data or programming.]

data mining
results due to time varying risk premia
some factor still missed
this is a dynamic strategy, so cannot be benchmarked to static loadings on FF factors.

3c.
We know dividend yields (D/P) are highly persistent over time. Many researchers have found that the market return can be described by the following regression model,

$$R_{mt} = a + b(D/P)_{t-1} + e_t \,.$$

Explain in words how it can be possible that returns are predictable, but that this predictability cannot be exploited to realize abnormal returns.

If the risk premia are time varying, a predicted higher return may just be a compensation for higher risk at a certain moment in time, or a higher price of the same risk.

3d.
Describe a setting where such predictability can be exploited.

If risk premia are constant and volatilities are constant etc, in short, if it is a real anomaly.

3e. (bonus)
Assume the correct discount factor is *M*, but you run the regression of excess returns of your portfolio on the excess returns of the stock index and a constant term. You obtain a significant alpha (constant term). Show using the OLS expressions for your estimators in your regression that this alpha may be spurious if the regression errors of your portfolio on *M* and a constant are correlated with those of the market on *M* and a constant.

Let $R = R_m,\ y = R_i$, then $R = c + Md + \eta,\ y = \alpha + M\beta + \varepsilon$. As the result is spurious, $\alpha = 0$.

The regression model is $y = a + Rb + e$. So

$$a = \bar{y} - \bar{R}\frac{\text{cov}(R, y)}{\text{var}(R)} = \bar{y} - \left(c + \bar{M}d\right)\frac{d\,\text{cov}(M, y) + \text{cov}(\eta, y)}{d^2\sigma_M^2 + \sigma_\eta^2} =$$

$$\bar{y} - \bar{M}\beta + \bar{M}\beta - \left(c + \bar{M}d\right)\frac{d\beta\sigma_M^2 + \sigma_{\varepsilon\eta}}{d^2\sigma_M^2 + \sigma_\eta^2} = \alpha + \bar{M}\beta - \left(c + \bar{M}d\right)\frac{d\beta\sigma_M^2 + \sigma_{\varepsilon\eta}}{d^2\sigma_M^2 + \sigma_\eta^2} =$$

$$\alpha + \bar{M}\left(\frac{\beta\sigma_\eta^2 - d\sigma_{\varepsilon\eta}}{d^2\sigma_M^2 + \sigma_\eta^2}\right) - c\frac{d\beta\sigma_M^2 + \sigma_{\varepsilon\eta}}{d^2\sigma_M^2 + \sigma_\eta^2}.$$

This can be unqual to α even if there is no correlation and no η, but $c \neq 0$.

Also if there is no correlation, $c = 0$, but $\sigma_\eta^2 > 0$, you do not recover the true alpha.

If there is correlation, it is obvious also that the recovered/estimated alpha may not be the true one.

4a.
There has been a long debate on whether the young should invest more in stocks than the old. One explanation has been that young people "play the game" of annually investing in stock more times than do old people, and therefore diversify their risk over time.
Provide to competing explanations why young people should invest more in stocks than old people.

labor income
hedging investment opportunity sets
mean reversion

4b.
Assume you have estimated the factor sensitivities of a number of excess returns using the regressions

$$R_{it} - r^f = \alpha + \beta_{i1} f_{1t} + \ldots + \beta_{iK} f_{Kt} + e_{it} = \beta_i' f_t + e_{it} \,,$$

where the f_t is the vector with factors (e.g., the Fama-French-Carhart factors) and β_i the vector of factor sensitivities for asset *i*. Some of your assets have significant alphas. You decide to make a portfolio with weight w_{it} in asset *i* at time *t* to take *maximum* advantage of the alphas *without* incurring any systematic risk. You are not allowed to take short position. Give the formulas for this optimization problem, i.e., the objective and the constraints.

$\max w'\alpha$

s.t.

$w_1\beta_{ik} + \ldots + w_n\beta_{nk} = 0$ for all $k = 1 \ldots K$

$w_1 + \ldots + w_n = 1$

$w_i \geq 0$ for all $i = 1 \ldots n$

4c.
You have the 2 factor model

$$R_{it} - r^f = \alpha + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + e_{it} = \alpha + \beta_i' f_t + e_{it} \,.$$

Factor 1 has mean 2% and standard deviation 8%, factor 2 has mean 4% and standard deviation 15%. The following numbers hold.

Security type	β_{i1}	β_{i2}	stdev(e_{it})
1	0.25	-0.50	12%
2	-0.50	0.50	15%
3	1.00	1.00	18%

There are 300 securities of each type. Show that an equally weighted portfolio of 160 type 1 and 120 type 2 stocks and 20 type 3 stocks has a zero systematic risk exposure.

160 * 0.25 + 120 * (-0.50) + 20 * 1 = 0
160 * (-0.50) + 120 * 0.50 + 20 * 1 = 0

4d.
What is the Sharpe ratio of this portfolio under (4c).

As stated in 4c, there is no beta, so no systematic risk in this portfolio.
So the Sharpe ratio is given by:

$$\sigma^2 = \frac{160}{300^2}0.12^2 + \frac{120}{300^2}0.15^2 + \frac{20}{300^2}0.18^2$$

$$SR = \left(\tfrac{160}{300}\alpha_1 + \tfrac{120}{300}\alpha_2 + \tfrac{20}{300}\alpha_3\right)/\sigma$$

4e.
Give another portfolio that also has a zero systematic risk exposure.
How can you make a choice between this new portfolio, and that of (4c) (or any other zero beta portfolio)?

Just cut the portfolio in half (or increase it by 50%) or ... many other possibilities. Compare by for example Sharpe ratio.