SOLUTIONS DECEMBER 2005

QUESTION 1

a) $E(R_i) = rf + \alpha_i + \beta_i\, MRP + \gamma_i\, VRP$, with MRP= market risk premium and VRP= value risk premium.
$Var(R_i) = \beta_i^2 . \sigma^2(M) + \gamma_i^2 . \sigma^2(V) + \sigma^2(\varepsilon)$

b) That beta of S1-S5 is zero follows directly from the underlying linear factor model. For any subportfolio Sj this model is

$R(S_j) = \alpha_j + \beta_j . R_m + \gamma_j . HML + \varepsilon_j$, with $\beta_j=1$ and $\varepsilon_j=0$

Subtracting S5 from S1 gives the result.

The absolute risk of R(Sj) and of R(S1-S5) follow directly from the above

F actormodel: the formula is completely similar to the variance-formula for Ri, given under a. The lower risk for R(S1-S5) is due to the fact that this portfolio has zero-exposure to the market risk factor. The remaining risk can be attributed to risk of the value factor itself.

c) STD (V) equals in this approach the standard-deviation of the mimicking portfolio S1-S5 and

is thus equal to 0.10. The variance of R(S2) can be written as $Var(S2)= \sigma^2(M) + (0.5)^2 . \sigma^2(V)$ or

$(0.22)^2 = \sigma^2(M) + 0.25. (0.10)^2$.

d) $Var(R_p-R_m) = (0.9-1)^2 . 0.2^2 + 1.5^2 . 0.10^2 = 0.0229$ so that $STD(R_p-R_m)= 0.1513$

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e) The active risk of portfolio S1-S5 follows from writing out the formula for the difference

$R(S1\text{-}S5) - R_m = -1.R_m + 1. R(S1\text{-}S5)$ The variance of this term is the sum of the variances:

$0.2^2 + 0.1^2 = 0.05$ so that the active risk (=st-dev) equals 0.2236. The total risk of portfolio

S1-S5 was already given and equals 0.10.

For a hedge fund the total risk measure is the most appropriate.

QUESTION 2

A) The manager has to run a cross section regression of the excess stock returns of month t on the computed g-values per stock in month (t-1). The relevant variable is the regression coefficient which has to be significant and positive. The regression analysis can be used to compute the correlation coefficient between the forecasting variable g and the excess stock return. This correlation-coefficient is also known as the information coefficient (IC). The IR (=Information Ratio) of this strategy depends on this IC and the number of independent bets you can make. Since you have a large number of stocks this number is quite high which tells you that IR will sufficiently high even if the IC on the level of the individual stocks is rather low.

B) Var(Rp)= w'XFX'w +w'Δw

C) The relative risk of portfolio P is the arithmetic average of the 100 individual residual risk terms ε_i. Thus the relative risk can be computed as the st=deviation of the arithmetic average of ($\varepsilon_1 \ldots \varepsilon_{100}$) which equals (1/10). $\sigma(\varepsilon) = 0.10/10 = 0.01$.

The relative risk of Q can be seen as the difference of two arithmetic averages which by nature are uncorrelated. The variance of the difference is in that case the sum of the variances which yields

$$(0.01)^2 + (0.01)^2 = 0.0002$$

So that the relative risk (=st-deviation) of Q is the

SQRT(0.0002) = 0.014

The difference in relative risk between P and Q lies in the fact that Q has a higher absolute exposure to individual residual stock risk.

D) A stratified technique selects the stocks with the highest alpha within each industry and then forms a portfolio which is industry-neutral. (Industry-neutral means that the industry weights in the portfolio are equal to the weights in the market portfolio.) Active risk in this technique is managed by diversification within industries and by avoiding exposure to systematic risk-factors by the industry-neutral constraint.

A quadratic optimization technique would have a quadratic target function, e.g. Alpha minus lambda times the variance of the difference in returns between the active portfolio and the market portfolio. You have to maximize this objective by finding the weights wi of the individual stocks; in practice you will impose constraints on these weights, e.g. all wi should be positive.

Controlling the tracking error in the stratified technique can be done by choosing the appropriate degree of diversification within each industry; reducing the tracking error would require more stocks per industry and/or choosing the weights of individual stocks more in line with their market weights. In a quadratic optimization framework the easiest way is to adjust the objective function to:

"maximize alpha subject to the constraint that the tracking error is smaller than 0.03".

E)

Alpha Source	Relative risk	Absolute Risk
Fundamental analysis (valuation)	Long only relative to market portfolio	long-short (hedge fund)
Controlling factor Exposures	Long only relative to market portfolio	long-short (hedge fund)
Tactical bets on assets	Small bets relative to Benchmark allocation	absolute risk strategy

Question 3

Part a.
Variance pf: 0.25*0.0933+2*0.25*-0.0899+0.25*0.0869 = 0.0001
Covariance: 0.5*0.0017 – 0.5*0.0015 = 0.0001
Correlation: 0.0001 / sqrt(0.0001 * 0.0001) = 1 = 100%

Part b.
$r^S = r^a - 0.5\, r^L$ in this case where A/L=2.
Max 0.08x + 0.06(1-x) – 0.1 * (0.0933 x^2 - 2 x (1-x) 0.0899 + $(1-x)^2$0.0869 - 2 x 0.5*0.0017 + 2 (1-x) 0.5 * 0.0015) →
0.02 = 0.1 * 2 * (0.0933 x – (1-2x) 0.0899 – (1-x) 0.0869 – 0.0016) →
0.01/0.1 = (0.0933 + 0.0869 + 0.1798)x – (0.0899 + 0.0869 + 0.0016) →
0.1 = 0.36x – 0.1784
X = 77.33%

Part c.
Part a shows that holding the 50%-50% mix, we have perfect correlation with the liabilities. So the 50-50 mix must be proportional to r^L. The 50-50 mix has a return variance of (0.0933 – 2*0.0899 + 0.0869)/4 = 0.0001.
So if we combine the 50-50 portfolio return r^p with the riskfree asset, we only need to construct the investment such that the surplus return has zero variance (we cannot get lower than that !) [i.e. leveraging up or down the 50-50 return to match the variance of -$0.5r^L$]
Surplus return variance is now given by [for leverage factor b]
$0.0001*b^2$ – 2*0.5*b*0.0001+0.25*0.0001 = 0 → b = 0.5
So half the assets in the riskfree asset, and the other half in the 50/50 mix.

Part d.
((0.4*0.08 + 0.6*0.06) – 1.645 $(0.16*0.0933 – 2 * 0.24 * 0.0899 + 0.36 * 0.0869)^{1/2}$) = -2.3%
So VaR = 2.3%*1M = €23,000.

Part e.
If you graph the pay-off of the portfolio, it is easy to see that the worst outcomes are in the middle. The 90% best outcomes are such that 45 left, 45 right, meaning that payoff at S=94 and S=106 are crucial. Payoffs there are €6, so a loss of <u>€2</u> with respect to the initial price of €8.

QUESTION 4.

Part a.
$(a+b(E/P)_T - 0.005) / (10\sigma^2)$

Part b.
Expected return unchanged, but variance is now
$\sigma^2(1 + T^{-1} + T^{-1}((E/P)^2/s^2))$, so $(a+b(E/P)_T - 0.005) / (10\sigma^2(1 + T^{-1} + T^{-1}((E/P)^2/s^2)))$

Part c.
Without parameter uncertainty: more in risky asset.
With: depents on E/P; if large, than out of the risky asset, but if small, than more into risky asset. The idea is that large E/P increases the expected return, but also the variance because of the uncertainty in the parameter estimate b. If E/P rises far, then the uncertainty component dominates because E/P enters quadratically rather than linearly as in the expected return.

Part d.
1-month: myopic result; but long term, then stock hedges against IOS changes as in Barberis. This means if the stock does poorly, reinvestment opportunities are very good (because in that case E/P has risen and thus expected future returns are higher). So the long term investor can go more into stock. If you also account for parameter uncertainty, the effect is mitigated, because the variance is increased.

Part e.
Dynamic investment strategies can be used to mimic options. So under these preferences, we will then get the casino effect out. We will follow a dynamic strategy paying out in cheap states of the world at the expense of large deficits in expensive states of the world. If not mitigated, this will be the main driving force in terms of incentives of the regulatory framework, which appears economically unhealthy.