Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration

Program:	M.Sc. Finance
Exam:	Investments 4.1
Program:	M.Sc. Finance, period 4.1
Vakcode:	60412040
Date:	October 28, 2004
Tijd:	15:15 – 18:00
Duration:	2 hours, 45 minutes
Parts:	The exam has 4 questions. The first two questions (1 and 2) are for the part taught by prof. Frijns. The second two questions (3 and 4) relate to the part taught by prof. Lucas. Each question makes up 25% of your grade for this written exam. Perhaps redundantly: the written exam makes up 70% of your final grade. The remaining 30% is scored by the cases done during the lecture period.
Score:	Each of the four questions in the exam yields you a maximum of 10 points. Your total score cannot exceed 40 points. The grade for this written exam is obtained by dividing the points scored by 4.
Results:	Results will be made known as soon as possible, but at the latest November 11, 2005.
Inspection:	You can inspect your marked exam papers Tuesday November 15, 9:00am. The room will be announced via the monitor system.
Remark:	Provide complete answers (including computations where appropriate). Always provide motivation/explanation of your answer, even if this is not mentioned explicitly in the question. A short 'yes' or 'no' will never do as an answer. Use your time efficiently.

Please answer Questions 1+2 on a separate sheet from Questions 3+4.

Good luck!

This document has 7 pages. Provisional answers in blue.

QUESTION 1

A portfolio manager for a traditional investment house tries to innovate by offering his active portfolio in different forms to the clients of the house. The existing active portfolio P consists of only 36 stocks where stocks are selected based on fundamental analysis of the underlying companies. The portfolio construction process is bottom-up: the 36 most promising stocks are equally weighted.

Part a
Compute the residual or relative risk, $\sigma(\varepsilon_p)$ of the portfolio relative to the market portfolio, assuming that the residual risk $\sigma(\varepsilon)$ of each individual stock is 0.36.

$P=\Sigma\ Ri/36$ so that $\sigma(\varepsilon p)= 0.36/\sqrt{36} = 0.06$

The beta of this portfolio, measured over the last 60 months, is equal to one and the alpha is a meagre 0.02.

Part b
What is the information-ratio of this strategy and compute the optimal relative risk for this portfolio, assuming that the clients have the following trade-off between alpha and relative risk of an active portfolio:

$U=alpha-\lambda\cdot\sigma(\varepsilon_p)^2$

with $\lambda=5$.

$IR=\alpha /\sigma(\varepsilon p) = 0.02/ 0.06 = 0.33$

$\sigma^* = IR/ 2\lambda = 0.33/ 10 = 0.033$ (=3.3%)

Part c
The optimal portfolio allows for a smaller relative risk level than that of the original portfolio P. A quick and dirty way to achieve this is to form a new portfolio MP which is a weighted average of the market portfolio M and the active portfolio P. Derive the weights of M and P if you want to halve the relative risk. What does this imply for the weights of the 36 originally selected stocks in the portfolio MP.

Mix portfolio P with an index portfolio (residual risk is zero). The weight of P in the new portfolio should be reduced to 0.5. The weights of the original 36 active stocks reduce to half their original weight of 1/36 plus half their weight in the market or index portfolio.

The manager decides to market his portfolio as an absolute return portfolio.

Part d
Compute the expected return, the total risk, $\sigma(p)$, and the Sharpe ratio, defined as "expected return/ $\sigma(p)$", of the original portfolio if the following characteristics of the market portfolio are known:

Risk free rate	= 0.03,
Market risk premium	= 0.03,

Market risk $\sigma(m)$ = 0.20.

$Rp = 0.02 + Rm + \varepsilon p$ so that
$E(Rp) = 0.02 + 0.06 = 0.08$ and
$\sigma^2(p) = \sigma^2(m) + \sigma^2(\varepsilon p) = 0.04 + 0.0036 = 0.0436$
$Sharpe(P) = E(Rp) / \sigma(p) = 0.38$

Finally the manager has the plan to restructure his portfolio into a hedge fund. Therefore, he forms an equally weighted portfolio of the ten stocks with the highest alpha (alpha=0.06) and an equally weighted portfolio of the ten stocks with the lowest alpha (alpha= -0.05). Both portfolio's have a beta equal to one. The hedge portfolio is long in the first and short in the second portfolio.

Part e
Give the formula for the hedge fund return. This formula describes the return as a linear function of factor returns and the noise factor. Compute the expected return, the standard deviation and the Sharpe ratio of this hedge fund.

First determine the return equations of the subportfolio's H1 and H2:
H1: $Rh1 = 0.06 + Rm + \varepsilon(H1)$, with $\varepsilon(H1) = \Sigma\varepsilon i / 10$ $i = 1,\dots,10$
H2: $Rh2 = -0.05 + Rm + \varepsilon(H2)$, with $\varepsilon(H2) = \Sigma\varepsilon i / 10$ $i = 27,\dots,36$
And than define the hedge portfolio H as
$H = H1 - H2$ or
$H = 0.11 + \varepsilon(H)$ (note that beta of H is zero)
$\sigma^2(\varepsilon H) = (0.11)^2 + (0.11)^2 = 0.0256$
so that
$Sharpe(H) = 0.11 / \sqrt{0.0256} = 0.11 / 0.16 = 0.69$

After some disappointing years a new manager decides to "pimp up" the portfolio by switching to a factor oriented strategy. He has identified two factors with a significant risk premium, the value factor V and the Momentum factor Mom. The characteristics of these factors are as follows:

	Exp return	standard deviation
V	0.02	0.10
Mom	0.01	0.15

The two factors are uncorrelated with each other and with the market return.
He constructs a portfolio Q with the following exposures to the three factors.

Factor	Exposure
Market	0.9
V	1.0
Mom	1.0

The residual risk of this broadly diversified portfolio is zero.

Part f
Write down the formula for the active risk of Q. Active risk is defined as the standard deviation of the difference (Q-M). Compute the active risk of Q.

$Q = 0.9\, M + V + Mom$ (note that $\varepsilon(Q) = 0$)

$Q - M = -0.1\, M + V + Mom$ so that

$\sigma^2(Q-M) = 0.01 \times 0.04 + 0.01 + 0.0225 = 0.0329$

so that active risk of Q is

$\sigma(Q-M) = 0.18$

QUESTION 2: CONCEPT CHECKS

The CART-model is a forecasting model for equity markets based on two fundamental risk factors, the real short rate and the term premium, and a valuation factor, the dividend yield. Based on the value of these factors the model predicts the likelihood that stocks will outperform cash in the next month.

Part a

Use the Levis-Liodakis approach to argue that the Information Ratio of an active timing strategy based on the CART model cannot be very high.

$IR = IC \times \sqrt{BR}$ with IC the correlation between forecast and realized return. The CART model forecasts only the "hit ratio", given the forecast that equities will outperform cash the actual out-performance can vary wildly. We can be confronted with the right forecast leading to a small out-performance and the wrong forecast resulting in a huge underperformance. As a result of this, the correlation coefficient between forecast and realization (=IC) cannot be very high and is considerably lower than the high hit-ratio's might suggest. Moreover the "breadth" of this strategy is rather small; the number of independent "bets" per year is only 12.

Part b

Assume that you want to implement an active sector-allocation strategy based on estimates of the implied risk premiums per sector. Explain how you would estimate these implied risk premiums. From practical experience you know that the raw estimates can vary wildly. How can you validate the appropriateness of this alpha-source and what technique could you apply to refine the raw forecasts

We can derive the implied risk premium from the dividend discount model, more specific the Gordon formula for the constant-growth case. This formula states that $P = D/(r+x-g)$ or after rewriting $d = r + x - g$, with $d = D/P$. So that the implied risk premium $x = d + g - r$.

The estimates for x for the individual sectors can vary wildly; in such cases in makes sense to make an outlier correction by shrinking the estimates to their overall mean. If x(i) is the estimated risk premium for sector i and x(average) the average of all sectors than the refined estimate for sector i can be found from

$$\tilde{x}(i) = x(i) + a.(x(average) – x(i))$$
$$\text{with } 0 < a < 1$$

Part c

You are told that the following factor model is a good candidate for an active strategy:

$R_i = \alpha + \beta \cdot R_m + \gamma \cdot Size + \delta \cdot CFROI + \varepsilon_i$

where CFROI, Cash Flow Return On Investments, measures the annualized return on past investments in firm i. Form a mimicking portfolio for the CFROI-factor and indicate how to estimate the mean and the standard deviation of this stochastic factor. Furthermore explain how you would determine the exposure of stock i to this factor

The mimicking portfolio for the factor CFROI can be formed by forming two sub-portfolio's, the first one consists of the 10% stocks with the highest CFROI and the second of the 10% stocks with the lowest CFROI. The mimicking portfolio is long in the first one and short in the second one. This long-short portfolio has small or zero exposures to all systematic risk factors except for the CFROI factor. The residual risk is also small, provided that the number of stocks in each sub-portfolio is sufficiently large. The returns of this long-short portfolio can than be seen as the returns on the factor CFROI, a sample of n consecutive returns can be used to estimate the mean return and the standard deviation.

Given the way the CFROI-factor is constructed the best way to estimate the exposure of an individual stock on this factor is to run a time series regression analysis of the stock returns on the factor returns (= returns of the mimicking portfolio). The slope of the regression equation measures the exposure.

There are three stocks, R1, R2, R3 and we form a portfolio P as follows

$P = w'R,$
$w' = (0.4, 0.4, 0.2)$
$R = X\beta + \varepsilon$

with X a three times two matrix of factor exposures and β the two times one vector of factor returns, and ε a three times one vector of residual risk factors.

Part d

Give the formula for P in terms of factor exposures, factor returns and residual risk. If the first column of X consists of ones and the elements of the second column are all equal to 0.5, compute the factor exposures β_1 and β_2 of P to the first and the second risk factor.

$P=w'R = w'X\beta + w'\varepsilon = Xp.\beta + \varepsilon p$ where Xp is a 1x2 row vector of Factor-exposures of portfolio P to the two factors β1 and β2 and εp is equal to $0.4 \times \varepsilon 1 + 0.4 \times \varepsilon 2 + 0.2 \times \varepsilon 2$.

The vector Xp equals in that case Xp= (1 , 0.5)

Part e
An active manager tries to impress you with his results. Having followed the courses in Investments, you have learned to be skeptical. You use the return data over the last 60 months of this portfolio to perform a regression analysis, which has the following specification

$$R(t)= \alpha + \beta \cdot R_m(t) + \gamma \cdot V(t) + \varepsilon(t)$$

where R(t) is the portfolio return, $R_m(t)$ the market return and V(t) the return of a Value factor. The results are as follows

	Estimate	significant
α	-0.03	yes
β	1.2	yes
γ	-0.3	no

Explain the results.

The out-performance comes from a high beta strategy and some exposure to Growth, that is exposure to systematic risk factors in excess of the exposure of the market portfolio (=benchmark portfolio). Adjusting for these risks we find that the true out-performance (=α) is negative. This under-performance is even statistically significant.

START THE LAST TWO QUESTIONS ON A NEW ANSWER SHEET SUCH THAT THE ANSWER SHEETS CAN BE SPLIT BETWEEN PROF. FRIJNS AND PROF. LUCAS FOR FASTER CORRECTION.

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QUESTION 3 (computational; ALM)

An insurance firm has the following balance sheet:

Assets		Liabilities	
Assets	140	Insurance Liabilities	100
		Equity	40

The Insurance Liabilities are random. There are two assets to invest in. The assets' returns and the returns on the liabilities follow a multivariate normal distribution with mean vector and covariance matrix specified as

$$\begin{pmatrix} r_1 \\ r_2 \\ r_L \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 5\% \\ 10\% \\ 2\% \end{pmatrix}, \begin{pmatrix} 1\% & 0.2\% & -0.6\% \\ 0.2\% & 4\% & 0.8\% \\ -0.6\% & 0.8\% & 4\% \end{pmatrix} \right)$$

where r_L is the insurance liability return, and r_1 and r_2 are the returns on asset 1 and 2, respectively.

Part a

Compute the correlations between asset 1 and 2, and between the assets' returns and the liability return.

(1,2): 0.2/sqrt(1*4) = 0.1 = 10%
(1,L): -0.6/sqrt(1*4) = -0.3 = -30%
(2,L): 0.8/sqrt(4*4) = 0.2 = 20%

This insurer will invest all his money in the two assets. For simplicity, we assume that short positions are allowed. Assume the position (denoted as a fraction) in asset 1 is equal to *x* and in asset 2 to (1-*x*).

Part b

Show that the portfolio return r_P and liability return are multivariate normally distributed with mean vector and covariance matrix as specified by

$$\begin{pmatrix} r_P \\ r_L \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 5\% \cdot x + 10\% \cdot (1-x) \\ 2\% \end{pmatrix}, \begin{pmatrix} 1\% \cdot x^2 + 0.4\% \cdot x \cdot (1-x) + 4\% \cdot (1-x)^2 & -0.6\% \cdot x + 0.8\% \cdot (1-x) \\ -0.6\% \cdot x + 0.8\% \cdot (1-x) & 4\% \end{pmatrix} \right)$$

(If you disagree with this result, then state what the result should be instead! Tip: write the portfolio return as a matrix product of the original returns.)

Note that

$$r_P = xr_1 + (1-x)r_2$$

$$\begin{pmatrix} r_P \\ r_L \end{pmatrix} = \begin{pmatrix} x & 1-x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_L \end{pmatrix}$$

The result now follows directly from the technical appendix, because

$$\begin{pmatrix} x & 1-x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5\% \\ 10\% \\ 2\% \end{pmatrix} = \begin{pmatrix} x \cdot 5\% + (1-x) \cdot 10\% \\ 2\% \end{pmatrix}$$

$$\begin{pmatrix} x & 1-x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1\% & 0.2\% & -0.6\% \\ 0.2\% & 4\% & 0.8\% \\ -0.6\% & 0.8\% & 4\% \end{pmatrix} \begin{pmatrix} x & 1-x & 0 \\ 0 & 0 & 1 \end{pmatrix}' =$$

$$\begin{pmatrix} x^2 1\% + (1-x)^2 4\% + 2x(1-x)0.2\% & -0.6\%x + 0.8\%(1-x) \\ -0.6\%x + 0.8\%(1-x) & 4\% \end{pmatrix}$$

Part c

If the insurer maximizes the expected portfolio return minus 4 times the portfolio variance, what is the optimal combination of assets 1 and 2 in that case.

Max -0.05x – 4*(0.01x^2 + 0.04$(1-x)^2$ + 0.004 x(1-x))
First order condition (derivative wrt x)
-0.05 – 4*(0.02x - 0.08(1-x) + 0.004(1-2x)) = 0
-0.05 – 4*(0.092x - 0.076) = 0
X = (-0.0125 + 0.076)/0.092 = 69%

Part d

Compute the liability hedging credit of the optimal portfolio computed under part c. Also give the interpretation of the liability hedging credit.

Covariance is -0.6% 69% + 0.8% 31% = -0.00166
L.hedging credit = 2*1*(100/140)*-0.00166 = -0.23%
For an uncorrelated portfolio with same variance, expected return has to rise by the liability hedging credit for a surplus optimizer.

Part e

Compute the optimal portfolio if instead the insurer maximizes expected surplus return (E[change in surplus/A_0]) minus 4 times the surplus return variance.
Explain the differences, or lack thereof, with your solution to part c.

Max -0.05x – 4*(0.01x^2 + 0.04$(1-x)^2$ + 0.004 x(1-x) - 100/140*(-0.012x + 0.016(1-x)))
-0.0125 = 0.02x – 0.08(1-x) + 0.004(1-2x) + 100 (0.012 + 0.016) / 140
X = (-0.0125 + 0.08 – 0.004 – (1.2 + 1.6)/140) / (0.02 + 0.08 – 0.008) = 47.3%

Much more now in asset 2, because this creates a larger positive correlation with the liabilities, and therefore a lower surplus volatility.

QUESTION 4 (digitals and concept checks)
[n.b.: patterns found in this question need not coincide with the patterns shown in class!!]

Assume the state of the world is summarized by a broadly diversified stock index. There is a number of traded derivative securities (digital options) with the following properties.

Price	Probability of ending in-the-money	Low strike	High Strike	Maturity
€ 5.98	6.12%	€0	€80	1 year
€15.91	16.23%	€80	€90	1 year
€ ????	25.33%	€90	€100	1 year
€23.65	24.13%	€100	€110	1 year
€15.52	15.83%	€110	€120	1 year
€12.12	12.36%	€120	∞	1 year
	100%			

Each digital can only be exercised at maturity (so it is European) and pays off €100 if it end up in-the-money (i.e., if the spot of the stock index one year from now lies between the low and the high strike value). The stock index is currently listed at €100, and a 1-year T-bill at €98.02.

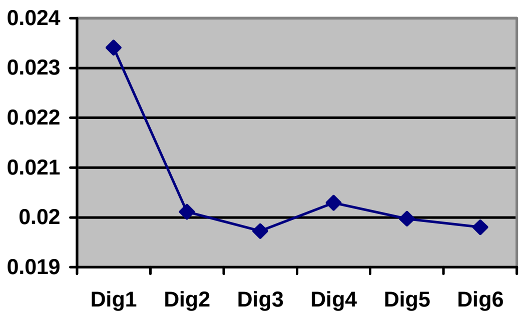
Part a
Compute the price of the digital with low strike €90 and high strike €100 given the information you have. Also explain in words why your answer is valid.

The digitals span the entire outcome of the stock. So buying all of them, should equal the price of the risk free asset with the same maturity. So the price is
98.02 – 5.98 – 15.91 – 23.65 – 15.52 – 12.12 = 24.84

Part b
Show (using the last two digitals from the table) that the price of an at-the-money call option (that entitles you to buy the index at €100) with a maturity of 1 year cannot have a price below €3.97. Explain your answer.
[If you do not agree with this claim, then show what the correct answer should be.]
[Tip: create a pay-off that is always lower than that of the call.]

Buying 0.10 of the penultimate digitals, and 0.20 of the ultimate one, gives you a payoff that is always below that of the ATM call. The price is 1.552 + 2.424 = 3.976. The ATM call always has a higher payoff than this portfolio of digitals, so its price must be higher.

Part c
Graph the expected returns of the digital options.



Part d
You currently have €1000. You cannot go short in any asset. Derive the optimal portfolio of digitals if you maximize your expected portfolio return subject to the 85%-confidence 1-year VaR of your portfolio not being greater than €10.
[Assume for simplicity that you can also buy fractions of options rather than an integer number of options.]

Because of the probability constraint, we can only drop digital 1 or 6. As 1 has the highest expected return, we drop 6 completely.
We by 9.9 of Digitals 2-5 (yielding 990 in settings 2-5), costing
9.9 * (15.91 + 24.84 + 23.65 + 15.52) = 791.208
The remainder 208.792 is invested in 205.792/5.98 = 34.41 digitals of the first type.

Part e
Critically discuss how Dert and Oldenkamp suggest to `partially solve' the Casino-effect. Explain their solution, and provide some motivated advantages and disadvantages of their solution.

See the paper.

Part f
Provide an example that shows that VaR is not a coherent risk measure, because it does not properly account for diversification effects in portfolios. Discuss in what sense this defect may be relevant or not.

The bond portfolio (concentrated or not) as treated in class and on the transparencies could be an example. This is clearly relevant if payoffs are non-normally distributed, as this dichotomous bond-payoff. For normally distributed payoffs, this is less relevant.