Exam Applied Stochastic Modeling
9 February 2015

This exam consists of **6** problems, each consisting of several questions. All answers should be motivated, including calculations, formulas used, etc. It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes. The use of a calculator is allowed.

For each question it is indicated in brackets how many points can be obtained. The grade for the exam is then given by: # points/$4 + 1$.

Exercise 1. The arrival process of customers is modeled as an inhomogeneous Poisson process. Between times 0 and 10 the arrival rate is λ and after time 10 the arrival rate is 0.5λ.
a. [2 pt.] Assume that it is now time 0. Give the probability that the first arrival occurs after time t, for every $t > 0$.
b. [2 pt.] What is the probability that exactly one customer arrives between times 0 and 20?

Exercise 2. Consider a machine with a lifetime X that is exponentially distributed. For its maintenance the machine is inspected every t time units. When the machine is down, it is immediately replaced by an identical new machine. For inspections of a working machine costs I are incurred. When the machine is down, costs d per unit of downtime are incurred.
a. [2 pt.] Determine explicitly the expected downtime between two inspections $\mathbb{E}(t - X)^+$.
b. [3 pt.] Use renewal theory to determine the long-run average costs per time unit for this maintenance policy.

Exercise 3. Consider a closed queueing network consisting of two single-server stations. Customers from queue 1 go to queue 2 and vice versa. Service times are assumed to be exponential and there are 3 customers in the system.
a. [2 pt.] Model this as a birth-and-death process.
b. [3 pt.] Calculate the stationary distribution of the number of customers at queue 1.
c. [2 pt.] Give the distribution of the number of customers at queue 1 as perceived by an arbitrary customers arriving at queue 1.

Exercise 4. Consider a web server farm consisting of two servers working according to FCFS. Arrivals follow a Poisson process with rate $1/b$ and job sizes S are uniformly distributed between 0 and b.
a. [2 pt.] Suppose jobs are randomly assigned to one of the servers. Show that $\mathbb{E}S = b/2$ and $\mathbb{E}S^2 = b^2/3$, and calculate the expected waiting time.
b. [3 pt.] Suppose that size-interval splitting is used and jobs of size smaller than t are considered small. Now, 'small' jobs are assigned to server one, whereas 'large' jobs are assigned to server two. Calculate the expected waiting time of 'small' and 'large' jobs.
c. [2 pt.] Explain what happens with the weighted-average waiting time compared to the answer of question a.

Exercise 5. A queueing network consists of three queues each with an infinite number of servers. New customers arrive to queue 1 with rate λ. After being served, they go to queue 2 with probability p; with probability $1-p$ they go to queue 3. After being served at queue 2, they go back to queue 1 with probability q; with probability $1-q$ they go to queue 3. After being served at queue 3, customers leave the system. The service times at the three queues are exponential with rate 3.
a. [2 pt.] Formulate the routing equations for this system. For which values of λ, p, and q is the system stable?
b. [3 pt.] Determine the expected total number of customers in the system in terms of λ, p, and q.
c. [2 pt.] Assume now that the service time at queue 3 has a general distribution with mean 1/3. Explain how the result of b is affected by this assumption, and give the marginal distribution of the number of customers at queue 3.

Exercise 6. A hotel has a capacity of C rooms. Business customers book late, but they are willing to pay 150 euros per room. Leisure customers book early but they are only willing to pay 100 euros per room. The demand of business customers is random (say D), whereas the demand of leisure customers is (at least) C. There is no revenue when a room remains unbooked.
a. [2 pt.] Give an expression for the expected revenue when S rooms are reserved for business customers.
b. [2 pt.] Determine the optimal booking limit S^*, i.e., the number of rooms that need to be reserved for business customers to maximize revenue.
c. [2 pt.] Assume now that the demand of leisure customers is also random, where the probability that the demand is less than C is positive. Explain what happens with the optimal booking limit S^* and the expected revenue.