Exam Applied Stochastic Modeling
15 December 2014

This exam consists of **6** problems, each consisting of several questions. All answers should be motivated, including calculations, formulas used, etc. It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes. The use of a calculator is allowed.

For each question it is indicated in brackets how many points can be obtained. The grade for the exam is then given by: # points/4 + 1.

Exercise 1. The arrival process of customers is modeled as an inhomogeneous Poisson process. Between times 0 and 5 the arrival rate is λ and between times 5 and 10 the arrival rate is 2λ. Assume that customer n arrives at some time t.
a. [2 pt.] What is the probability that the interarrival time between customers n and $n+1$ is larger than 3 when $t = 1$? And what if $t = 3$?
b. [3 pt.] Customers remain in the system for exactly 2 time units. What is the mean number of customers in the system at time 6? And what is its distribution?

Exercise 2. Consider the production time S of a single production run. The acceptable production time is τ. After time τ, costs are incurred per unit of excess, i.e., the costs for a single production run are equal to $(S-\tau)^+$. Assume that S is exponential. After each production run, there is a set-up time of exactly 1 time unit, after which a new production run starts.
a. [3 pt.] Determine explicitly the expected costs per production run $\mathbb{E}(S-\tau)^+$.
b. [2 pt.] What are the long-run average costs per time unit?

Exercise 3. Consider an M/G/1 queue with arrival rate 1/3 and the service time S is equal to 1 with probability 3/4 and equal to 3 with probability 1/4. Customers with service time 1 are denoted by 'small' and customers with service time 3 are denoted by 'large'.
a. [2 pt.] Show that $\mathbb{E}S = 3/2$ and $\mathbb{E}S^2 = 3$, and calculate the expected waiting time.
b. [3 pt.] Calculate the expected waiting time of 'small' and 'large' customers when 'small' has (nonpreemptive) priority.
c. [2 pt.] Explain what happens with the weighted-average waiting time compared to the answer of question a.

Exercise 4. Consider a machine repairman model with N machines and a single repairman (also known as the Engset delay model $M/M/1/N/N$). Thus, up times and repair times are assumed to be exponential. Machines fail independently and there are sufficient waiting places at the repair facility.

a. [2 pt.] Model this as a birth-and-death process.

b. [3 pt.] Calculate the stationary distribution of the number of machines at the repair facility.

c. [2 pt.] Give the distribution of the number of machines at the repair facility as perceived by an arbitrary arriving machine.

Exercise 5. A queueing network consists of two queues with each a single server. New customers arrive to queue 1 with rate λ. After being served, they go to queue 2 with probability p; with probability $1-p$ they go back to queue 1. After being served at queue 2, they leave the system. The service times at both queues are exponential with rate 1.

a. [2 pt.] Formulate the routing equations for this system. For which values of λ is the system stable?

b. [2 pt.] Give the stationary distribution of the joint number of customers, formulated in terms of λ and p.

c. [2 pt.] Assume $\lambda = 0.5$. Make a sketch of the expected waiting time at queue 1 as a function of p, and explain its behavior.

Exercise 6. Consider a continuous-review continuous-product deterministic inventory model with holding costs, order costs and zero lead time (the EOQ model).

a. [2 pt.] Give the cost function for arbitrary order size Q and give the optimal order size Q^* that minimizes the average costs per time unit.

We change the model as follows. When Q is ordered, then the order costs are $K+kQ$ instead of K.

b. [4 pt.] Give the new cost function and determine the optimal order size. Compared to Q^* of question a, can this optimal order size be explained?