Exam Applied Stochastic Modeling
10 February 2014

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give 2.25 points when correctly answered.
The use of a calculator is allowed.

1. Consider the $M|M|2|3$ queue.
a. Draw the state-transition diagram of the number of customers in the system.
b. Give its stationary distribution.
c. Give the distribution as it is perceived by an arbitrary arriving customer. Motivate your answer.
d. Give the distribution as it is perceived by an arbitrary departing customer.

2. Consider an $M|G|1$ queue with $\lambda = 1$ and the following service time distribution S: $\mathbb{P}(S = 0) = 2/3$, and with probability $1/3$ S is exponential with average 2.
a. Calculate $\mathbb{E}S$, $\mathbb{E}S^2$ and $\sigma^2(S)$.
b. Give a formula for the long-run average waiting time and calculate it.
Suppose now that customers with $S = 0$ do not enter the queue anymore.
c. Calculate the long-run average waiting time of customers who enter the queue and for all customers (counting 0 for customers with $S = 0$).

3. Consider an $M|G|s|s$ queue with a hyperexponential service time distribution with 2 phases, i.e., the service time is a random mixture of 2 exponential distributions. Model this system as a 2-dimensional continuous-time Markov chain.
a. Draw the state-transition diagram.
b. Compute the stationary distribution of this Markov chain for $s = 1$ and use this to determine the long-run expected fraction of time that the server is busy.
c. Compute the stationary distribution for $s = 2$.
d. Compute the long-run expected number of servers that are busy for $s = \infty$. Explain which theorem(s) you have used.

4. Consider a deterministic-demand continuous-review inventory model where we do not allow for backorders or lost sales.
a. Draw a typical time-inventory diagram and formulate the appropriate inventory model (the EOQ model) to compute the optimal order size.
b. Assume the following parameter values: order costs $K = 200$, holding costs $h = 1$ per day. Compute the optimal order size and the time between two orders for 3 different levels of demand: $\lambda = 1$, 2 and 4 per day.
c. Compute for all cases the minimal total costs per unit of time and per item.
Consider the minimal total costs per unit of time and the minimal total costs per item as a function of λ.
d. On the basis of the results for c, indicate whether you expect these functions to be increasing or decreasing and convex or concave.
e. Give an intuitive explanation of your findings.