Exam Applied Stochastic Modeling
17 December 2009

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. Each question gives the same number of points when correctly answered.
The use of a calculator is allowed.
A table of the Poisson distribution is attached.

1. Consider a single server with an infinite queue to which jobs arrive according to a Poisson process with rate 1. The service time distribution has density $f(x) = 2x$ if $x \in [0, 1]$, 0 otherwise. Jobs are processed in order of arrival.
a. Calculate the distribution function and draw its graph.
b. Calculate the first two moments of the service time distribution.
c. Calculate the expected long-run waiting time in this system.

2. Consider a flight with cheap and expensive ticket (costing 100 and 300 respectively). The demand for the expensive tickets is Poisson with average 3. There are no possibilities of buying back tickets.
a. Compute the optimal number of seats that should be reserved for customers willing to buy expensive tickets.
An intermediate class with price 150 is introduced. Bookings in this class are made before the high-paying customers, demand is Poisson with average 5. Now there are two reservation levels: one for the two most expensive classes, one for the most expensive class.
b. Prove that the sum of two independent Poisson distributions has again a Poisson distribution.
c. Determine both reservation levels as to obtain the highest possible revenue. Motivate the way you calculated the reservation levels.

3. A queueing network consists of two queues each with a single server. New customers arrive to queue 1 according to a Poisson process with rate λ. After being served they move to queue 2. With probability p they leave the system after being served at queue 2; with probability $1-p$ they go back to queue 2. The service times at both queues are exponential with rate μ_i for queue i.
a. Formulate the routing equations for this system. For which values of λ is the system stable?
b. Give the steady state probabilities of this system, formulated in term of λ, p, μ_1, and μ_2.

4. In a theme park customers arrive at a certain attraction according to a Poisson process. The attraction serves people in batches: when the kth arrives they all leaves the queue at once and start being served.
a. Model the dynamics of the waiting line as a regenerative process. Describe what the renewal points are.
b. Compute the long-run average waiting time.
c. Assume that on average 1 customer arrives per minute and that $k = 10$. Compute the probability that an arbitrary arriving customer has to wait longer than 5 minutes.

Table with value of P(X>k) with X with a Poisson distributed random variable with mean mu

	values of mu									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000