Exam Applied Stochastic Modeling
10 February 2009

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All question are equally counted.
The use of a calculator and a dictionary are allowed.
A table of the Poisson distribution is attached.

1. Consider a system with 2 machines and a repairman. Machines fail independently with rate λ. The repairman repairs machines at rate μ.
a. Model this system as a birth-death process.
b. Calculate the stationary distribution and use this to derive the long-run expected number of machines that are functioning.
c. Derive the long-run distribution at moments that a machine fails.
d. Use this to derive the distribution of the long-run average time a machine waits before it is taken into service and calculate its expectation.

2. Consider a continuous-time multi-order deterministic-demand continuous-product inventory model with $\lambda = 5$, $K = 10$, $h = 1$ and $L = 1$.
a. Compute the optimal re-order level and re-order size.
Now demand is stochastic; it occurs according to a Poisson process with rate $\lambda = 5$. For the rest the system is the same. Items that are not available are backordered.
b. We use the same re-order policy. Estimate the probability that backorders occur in a cycle.
c. It is the objective to avoid backorders in at least 9 out of 10 cycles. How should we choose the re-order policy to achieve this?

3. Consider an $M|G|1$ queue with arrival rate 0.5 and service time distribution $S = X + Y$, with X and Y independent and both exponentially distributed with rates 1 and 2, respectively.
a. Calculate $\mathbb{E}S$, $\mathbb{E}S^2$, $\sigma^2(S)$ and $c^2(S)$.
b. Calculate the expected waiting time and the expected sojourn time for the $M|G|1$ queue.
c. What is the probability that an arbitrary arrival finds an empty system?

4. Consider a homogeneous Poisson process on $[0, T]$ with rate λ.
a. What is the expected number of arrivals in this interval?
b. Let the first arrival occur at t. Conditioned on this event, what is the expected number of arrivals in $[0, T]$?
c. Calculate the expected number of arrivals in $[0, T]$ again, but now using the law of total probability and the answer found under b.
d. Repeat a, b and c for an inhomogeneous Poisson process.

Table with value of P(X>k) with X with a Poisson distributed random variable with mean mu

	values of mu									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000