Exam Applied Stochastic Modeling
15 December 2008

This exam consists of 4 problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give 2.25 points when correctly answered.
The use of a calculator is allowed.

1. Consider the $M|G|1|1$ queue, thus a single server with no waiting places. The service times are *hyperexponential*: with probability p the service time is exponential with parameter μ_1, with probability $1-p$ exponential with parameter μ_2. We assume $\mu_1 \neq \mu_2$.
a. Formulate the PASTA principle.
b. Use renewal theory to calculate the probability that an arrival is rejected.
c. Model this system as a continuous-time Markov chain and calculate its stationary distribution. Use two states to model the service time. Verify that the answers correspond to what is found under b.
d. Draw the state-transition diagram of the $M|G|1$ queue with the same service-time distribution.

2. Consider the $M|G|1$ queue with the service time distribution S hyperexponential as defined in question 1. The parameters are as follows: $\lambda = \mu_1 = 1$, $\mu_2 = 2$, and $p = 3/4$.
a. Calculate $\mathbb{E}S$ and $\sigma^2(S)$.
b. Calculate $\mathbb{P}(W_Q = 0)$.
c. Calculate $\mathbb{E}W_Q$.

3. Consider again the hyperexponential distribution of question 1.
a. Calculate its distribution function.
b. Calculate its density.
c. Calculate its hazard rate.

4. Consider a continuous-review continuous-product deterministic inventory model with holding costs, order costs and 0 lead time (the EOQ model).
a. Give the optimal order size S^*.
We change the model as follows. When S is ordered then a random amount is delivered which is uniformly distributed between 0 and S.
b. Calculate the average holding costs.
c. Calculate the optimal order quantity.