Faculty of Science	2nd partial exam Analysis II
Department of Mathematics	26-05-2021
Vrije Universiteit Amsterdam	12:15 - 14:30

The use of a calculator, a book, or lecture notes is not allowed.
Please, motivate your answers! A correct answer without proper motivation and calculations will yield no points.
Your hand-written answers need to be submitted through Canvas before 14:30, CEST, in a single PDF file.
You have 135 minutes to compleat the test.
The average of the first and second partial exams will be your final grade.

Question 1. [4 points] Prove or disprove the following statement:

"The set $A = \{(x_1, x_2) \in \mathbb{R}^2 : 3 < x_1 < 7, 2 \leq x_2 \leq 5\}$ is an open set."

Question 2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1^4 - x_1 x_2^2 + x_2^4}{x_1^2 + x_2^2} & \text{if } (x_1, x_2) \neq (0,0), \\ 0 & \text{if } (x_1, x_2) = (0,0). \end{cases}$$

(a) [4 points] Is f continuous at $(0,0)$?
(Answer without proof or disproof yields no point).

(b) [1 point] Calculate $\frac{\partial f}{\partial x_1}(0,0)$ and $\frac{\partial f}{\partial x_2}(0,0)$.

(c) [4 points] Is f (totally) differentiable at $(0,0)$?
(Answer without proof or disproof yields no point).

Question 3. Consider the equation

$$xyz + 2z\cos y + y \sin z = 2\pi.$$

(a) [2 points] Can the equation be solved for z in terms of x and y in a neighborhood of $(\frac{4}{\pi}, \pi, \frac{\pi}{2})$?
(Answer without proof or disproof yields no point).

(b) [3 points] Determine $\frac{\partial z}{\partial x}$ in a neighborhood of $(\frac{4}{\pi}, \pi, \frac{\pi}{2})$.

Question 4. [7 points] Find, if it exists, the minimum of $\frac{1}{x^2+y^2}$ subject to the constraint $5x^2 + 5y^2 + 6xy = 4$ using Lagrange multipliers.

Question 5. [4 points] Calculate the double integral

$$\iint_G ye^{x^2+3} dA$$

where G is the triangle with corner points $(0,1)$, $(1,0)$, $(1,2)$.

Question 6. [7 points] Calculate the double integral

$$\iint_S \frac{x_1 x_2}{(x_1^2 + x_2^2)^3} dA$$

where $S = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \geq 0, 0 \leq x_2 \leq x_1, x_1^2 + x_2^2 \geq 1\}$.