Faculty of Science	Midterm exam Analysis II
Department of Mathematics	26-03-2021
Vrije Universiteit Amsterdam	12:15-14:30 pm

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Find the solution to the following ODE with initial value:

$$\begin{cases} y'(x) + \frac{3}{x}y = \frac{2}{x^2} + e^{x^4}, \\ y(1) = 1. \end{cases}$$

2. Find the general solution of

$$y''(x) - 6y'(x) + 13y(x) = x + e^x.$$

3. Determine if the following series are convergent or divergent. If the series is convergent, explain if it is conditionally convergent or absolute convergent.

a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{(n^2+4n+2)}{2^n e^n}$, b) $\displaystyle\sum_{n=3}^{\infty}(-1)^n \frac{\ln(n)}{n}$.

4. The sequence of functions $\{f_n\}$ is defined by

$$f_n(x) = \frac{1}{n}x\sin(x) + x, n \geq 1,$$

   a) Prove that $\{f_n\}$ converges uniformly on any closed interval $[a, b] \subseteq \mathbb{R}$.
   b) Does $\{f_n\}$ converge uniformly on $\mathbb{R}$?

5. Shows that the following series is uniformly convergent on $\mathbb{R}$,

$$\sum_{n=0}^{\infty} \frac{x}{(n+x^2)^2}.$$

Please turn over

6. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(3x+1)^{4n}}{n\,16^n}.$$

a) Determine its interval of convergence.

b) Suppose that this power series converges to $f(x)$ on an open interval around 0. Determine $f'(0)$. [Explain all your steps!]

Scores:

1 : 4	2 : 5	3 : a) 3 b) 4	4 : a) 4 b) 4	5 : 6	6 : a) 3 b) 3
4	5	7	8	6	6

$$\text{Grade} = \frac{\#\text{ points}}{4} + 1$$