Vrije Universiteit Amsterdam | **May 29, 2020**
Analysis II - Final Exam | **12:15 - 14:30**

Please motivate your answers! A correct answer without proper motivation will yield no points. Your hand-written answers need to submitted through Canvas, before 14:30 CEST in a single PDF file.

You have 135 minutes to complete the test. The result of this test will be your final grade.

Question 1 For each of the series below, determine whether the series convergences or diverges. In the case of convergence, determine whether the series converges absolutely or conditionally. `Justify your answer!`

(a) [5 points] $\sum_{n=1}^{\infty} \binom{2n+1}{n-1}$

(b) [7 points] $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

Question 2 On the interval $[0, 1]$, the sequence of functions $f_n(x)$ is defined by

$$f_n(x) = x(1 - x^n), n \geq 1.$$

(a) [7 points] Show that $\{f_n(x)\}$ converges uniformly on $[0, a]$ for every $0 < a < 1$.

(b) [3 points] Determine if the sequence of functions is uniformly convergent on $[0, 1]$.

Question 3 [10 points] Calculate

$$\int_3^4 \sum_1^{\infty} \frac{1}{(n+x)^2}.$$

`Hint: First show that the series is uniformly convergent on` $[3, 4]$.

Question 4 Consider the power series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{n3^n}.$$

(a) [8 points] Determine the interval of convergence.

(b) [5 points] Let $g(x)$ be the sum of this power series in the open interval of convergence. Find $g'(-6)$.

`Continue on the next page!`

Question 5 Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined by

$$f(x_1,x_2)=\begin{cases}\dfrac{4x_1^7+x_1^4x_2^2}{(x_1^2+x_2^2)^{\frac{5}{2}}} & \text{if } (x_1,x_2)\neq(0,0),\\ 0 & \text{if } (x_1,x_2)=(0,0).\end{cases}$$

(a) [7 points] Is f continuous on $\mathbb{R}^2$?

(b) [8 points] Is f (totally) differentiable at $(0,0)$?

Question 6 [7 points] The function $f:\mathbb{R}^2\to\mathbb{R}$ has continuous second partial derivatives on $\mathbb{R}^2$. Let $z=f(x_1,x_2)$ with $x_1=\ln(u_1+u_2)$, $x_2=\ln(u_1u_2)$. Calculate $\frac{\partial z}{\partial u_1}$ and $\frac{\partial^2 z}{\partial u_2\partial u_1}$ in terms of u_1 and u_2 and partial derivatives of f.

Question 7 [7 points] Find the minimum of the function $f(x_1,x_2)=x_1^2+x_2^2+10x_1x_2$ subject to the constraint $4x_1^2-3x_1x_2+4x_2^2=110$ using Lagrange multipliers.

Question 8 (a) [8 points] Let $G=\{(x,y)\in\mathbb{R}^2\setminus\{(0,0)\}\mid x^2\leq y, 8x^2\geq y, x\leq 4y^2, x\geq 2y^2\}$. Calculate

$$\iint_G \frac{12}{x^4y}e^{\frac{1}{xy}}\,dA.$$

(b) [8 points] Let $D=\{(x,y)\in\mathbb{R}^2\mid x^2+y^2\leq 16, x\leq y\leq -x\}$. Calculate if possible,

$$\iint_D \frac{x}{(x^2+y^2)^{\frac{1}{8}}}\,dA.$$