Faculty of Science	2^{nd} partial exam Analysis II
Department of Mathematics	24-05-2019
Vrije Universiteit Amsterdam	12:00-14:45 uur

The use of a calculator, a book, or lecture notes is not allowed.
Do not just give answers, but give calculations and explain your steps.

1. Let $G = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 > 0, x_2 > 0, x_1 + x_2 < 1\}$.
   a) Give an accumulation point of G that is not interior.
   b) Is G open, closed, or neither? (Prove or disprove your answer).
   c) Which is the smallest compact set containing G? (Justify your answer).

2. Is the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1^5 + x_2^5 + x_1 x_2^3}{x_1^4 + x_2^4} & \text{if } (x_1, x_2) \neq (0,0), \\ 0 & \text{if } (x_1, x_2) = (0,0). \end{cases}$$

   continuous on $\mathbb{R}^2$?

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1^2 x_2^3}{x_1^2 + x_2^2} & \text{if } (x_1, x_2) \neq (0,0), \\ 0 & \text{if } (x_1, x_2) = (0,0). \end{cases}$$

   a) Calculate $\frac{\partial f}{\partial x_1}(0,0)$ and $\frac{\partial f}{\partial x_2}(0,0)$.
   b) Is f (totally) differentiable at $(0,0)$?

4. The function $f : \mathbb{R}^2 \to \mathbb{R}$ has continuous second partial derivatives on $\mathbb{R}^2$. Let $z = f(x_1, x_2)$ with $x_1 = u_1^2 u_2$, $x_2 = u_1 + 2u_2$. Calculate $\frac{\partial z}{\partial u_1}$ and $\frac{\partial^2 z}{\partial u_2 \partial u_1}$ in terms of u_1 and u_2 and partial derivatives of f.

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x_1, x_2) = \frac{1}{2 + x_1 - 2x_2}$. Determine the second order Taylor polynomial near $(2,1)$.

6. Consider the system of equations

$$\begin{aligned} x e^y + uz - \cos v &= 2 \\ u \cos y + x^2 v - y z^2 &= 1. \end{aligned}$$

   a) Prove that this system of equations can be uniquely solved for (u, v) in terms of (x, y, z) in a neighborhood of $(x, y, z, u, v) = (2, 0, 1, 1, 0)$.
   b) Determine $\frac{\partial u}{\partial z}(2,0,1)$ and $\frac{\partial v}{\partial z}(2,0,1)$.

Please, turn over

7. Find and classify the critical points of the function $f(x_1, x_2) = 2x_1^3 - 6x_1x_2 + 3x_2^2 + 37$.

8. Find the maximum and minimum of the function $f(x_1, x_2) = x_1^3 + 9x_1^2 + 6x_2^2$ subject to the constraint $x_1^2 + x_2^2 = 9$ using Lagrange multipliers.

9. a) Let $G = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y \leq 4, x \geq 0, y \geq 0\}$. Calculate $\iint_G xe^{8y-y^2}\,dA$.

   b) Let $D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq y^2 - x^2 \leq 4, y \geq 2x, x \geq 0\}$. Use a change of variables to calculate

$$\iint_D \frac{x}{y}\,dA.$$

Scores:

1	2	3	4	5	6	7	8	9
1 : a) 1 b) 2 c) 1	2 : 3	3 : a) 1 b) 3	4 : 3	5 : 3	6 : a) 1 b) 2	7 : 3	8: 5	9: a) 3 b) 5
4	3	4	3	3	3	3	5	8

$$\text{Grade} = \frac{\#\text{points}}{4} + 1$$