Faculty of Science	Midterm exam Analysis II
Department of Mathematics	29-03-2019
Vrije Universiteit Amsterdam	08.45-11.30 am

The use of a calculator, a book, or lecture notes is <u>not</u> permitted.
Do not just give answers, but give calculations and explain your steps.

1. Solve the initial value problem

$$\begin{cases} y(x)y'(x) = \cos x, \\ y(\pi) = 2. \end{cases}$$

2. Find the general solution of

$$4y''(x) + 4y'(x) + y(x) = x + e^x.$$

3. Determine if the following series are convergent or divergent. If the series is convergent explain if it is conditionally convergent or absolute convergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \ln n}$, b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{n^2}}{n!}$.

4. Determine whether the following statements are true or not. If the statement is true, give a proof. If it is not true, give a proof or provide a counter example.

   a) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then the series $\sum_{n=1}^{\infty} a_{2n}$ is also convergent.

   b) If the series $\sum_{n=1}^{\infty} b_n$ is divergent, then the series $\sum_{n=1}^{\infty} b_{2n}$ is also divergent.

5. The function $f(x)$ and the sequence of functions $\{f_n(x)\}$ are defined by

$$f_n(x) = \frac{x^2}{n}, n \geq 1, \qquad \text{and} \qquad f(x) = 0.$$

   a) Prove that $\{f_n(x)\}$ converges uniformly to $f(x)$ on $[-a, a]$ for any $a > 0$.

   b) Determine if $\{f_n(x)\}$ converges uniformly to $f(x)$ on $\mathbb{R}$.

Please turn over

6. Consider the series of functions

$$\sum_{n=0}^{\infty} xe^{-nx}$$

   a) Prove that this series converges uniformly on $[a, \infty)$, for every $a > 0$.
   b) Prove that this series is not uniform convergent on $[0, \infty)$.

7. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^{3n}}{n\, 8^n}.$$

   a) Determine its interval of convergence.
   b) Suppose that this power series converges to $f(x)$ on an open interval around 0. Determine $f'(0)$. [Explain all your steps!]

8. Find a power series representation of the function

$$\frac{5}{2-3x}$$

centered at $x = -1$, and determine the interval of convergence.

9. Determine the Maclaurin series, and the interval of convergence, of the function K defined by

$$K(x) = \int_0^x t^2 e^{5t^2}\, dt.$$

[Hint: Start with calculating the Maclaurin series for $t^2 e^{5t^2}$. Explain all your steps!]

Scores:

1 : 3	2 : 3	3 : a) 3	4 : a) 2	5 : a) 3	6 : a) 3	7 : a) 3	8 : 3	9 : 3
		b) 2	b) 2	b) 2	b) 2	b) 2		
3	3	5	4	5	5	5	3	3

$$\text{Grade} = \frac{\#\text{ points}}{4} + 1$$