Exam Advanced Simulation for Finance, Economics, and Busines

Tuesday 26 May 2020, 15:00-17:00

Exercise 1 [40 Credits]

Consider the price of a stock process at time T under the risk neutral measure:

$$S(T) = S_0 e^{(r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z},$$

where S_0, r, σ, T are positive real numbers, and where Z is a standard normal distributed random variable. Suppose that you are interested in the value of $J = \mathbb{E}[1\{S(T) > K\}]$ for some strike K, and that you apply Monte Carlo simulation for estimating J.

(a). **[10 Credits]** Give the expression of the sample average estimator using sample size n.

(b). **[10 Credits]** How do you estimate the standard error of the estimator?

(c). **[10 Credits]** How do you construct 95% confidence intervals?

(d). **[5 Credits]** Now write down the Monte Carlo algorithm for estimating J.

(e). **[5 Credits]** How would you verify your computer program (i.e, check that your code is correct)?

Exercise 2 [25 Credits]

Consider a social network having n agents. The trust relation between agents is modeled via a Markov chain P, and the vector of initial beliefs is denoted by c. In the follwing we consider variations of the DeGroot learning model.

(a). **[10 Credits]** Give a belief update rule that allows for agents to be stubborn (in the sense that they prefer to stick to their own opinion).

(b). **[10 Credits]** Give an update rule that models herding behavior (agents follow their "neighborhood" as much as possible).

Triangular closure is the tendency that your friends are likely to become friends as well. This principle can explain why connections that were not present at some time t are present at a time $t + h$, for $h > 0$.

(c). **[5 Credits]** Consider a social network having n agents. Let $X_{ij} = 1$ if there is an arc from i to j in graph $\mathcal{G}$ observed at time t and zero otherwise. Suppose that you would inspect the social network at some later time, say, $t + h$. Give a formula for identifying the arcs (k, l) that you expect to appear at time $t + h$ due to triangular closure.

Exercise 3 [35 Credits]

A batch-job computer facility with a single CPU opens its doors at 7 A.M. and closes its doors at midnight, but operates until all jobs present at midnight have been processed. Assume that jobs arrive at the facility with i.i.d. interarrival times having distribution function F_A and mean $1/\lambda$. Jobs request either express (class 4), normal (class 3), deferred (class 2), or convenience (class 1) service; and the classes occur with respective probabilities 0.05, 0.5, 0.3 and 0.15. When the CPU is idle, it will process the highest-class job present, the rule's being FIFO within a class. The times required for the CPU to process class 4,3,2, and 1 jobs are independent random variables with respective distribution functions F_4, F_3, F_2 and F_1 and respective means $1/\mu_4, 1/\mu_3, 1/\mu_2$, and $1/\mu_1$. A job being processed by the CPU is not preempted by an arriving job of a higher class (this means: the service of the job is completed before the CPU starts a new job).

The objective is to estimate for $j = 1, 2, 3, 4$:

- ℓ_j: expected average number of class j jobs in queue per day;
- d_j: expected average delay of class j jobs in queue per day;
- f_j: expected fraction of CPU busy time spent on class j jobs per day.

(a) **[10 Credits]** Consider d_1. What (outcomes of) random variables do you need to observe to estimate d_1? Give the formula of the estimator to be used. Do the same for f_1.

(b) **[10 Credits]** Suppose that you construct a discrete-event simulation model (DESM) for estimating the performance measures. Specify the events and the event list, the system state of your DESM that enable you to observe the system dynamically in time, and define the counter (or statistical) variables to be used for calculating the output variables for the performance measures.

(c) **[10 Credits]** The (pseudo-)code of simulating a single day looks like

```
t = 0;
while t<17
    [t,event] = schedule_next_event;
    if event = arrival : ???;
    if event = departure : ???;
end;
complete_until_empty;
```

Time is measured in hours and t represents the current simulation clock time. Give the details of how you need to update state, event list and counter variables in the two procedures indicated with ???-marks.

(d) **[5 Credits]** How would you verify your program?