1. **Savings rate in the Ramsey-Cass-Koopmans model (20 points)**

   In this problem we consider the steady-state savings rate in the Ramsey-Cass-Koopmans model. The equilibrium dynamics of consumption per efficiency unit of labor, c, capital per efficiency unit of labor, k, and output per efficiency unit of labor, y, in the Ramsey-Cass-Koopmans model are given by the differential equations

$$\frac{\overset{\bullet}{c}}{c} = \frac{\left(\frac{1}{k}\right)^{1-\alpha} - \delta - \rho - \theta g}{\theta} \tag{1}$$

$$\frac{\overset{\bullet}{k}}{k} = \left(\frac{1}{k}\right)^{1-\alpha} - (n+g+\delta) - \frac{c}{k} \tag{2}$$

   and

$$y = k^{\alpha}. \tag{3}$$

   The parameters δ, g, θ, α, and ρ all have the same interpretation as discussed in class, in Romer, and in the slides. Throughout this question, we consider the steady-state values of these variables on the balanced growth path.

   (a) (4 points) The steady-state values, c^*, k^*, and y^*, are constant on the balanced growth path. At which rate do actual consumption, capital, and output (all per capita) grow on this path?

   (b) (4 points) Use equation (1) to solve for the steady-state capital stock, k^*, in this economy.

   (c) (4 points) How does the steady-state capital stock, k^*, depend on the discount rate δ? Is it increasing or decreasing? Use (1) to explain why.

   (d) (4 points) In the Solow model the savings rate, $s = i/y = (y-c)/y$, is taken as fixed and constant. In the Ramsey-Cass-Koopmans model this rate is determined endogenously as a function of the model parameters on preferences and technology. Solve for the steady-state savings rate implied by equations (1) through (3).

   (e) (4 points) How does the steady-state saving rate depend on the depreciation rate δ? Explain why.

2. **The intertemporal elasticity of substitution (20 points)**

   In class, we solved the real business cycle model with log preferences, such that the utility function was

$$U\left(C_t, L_t^s\right) = \ln C_t + b \ln\left(1 - L_t\right) \tag{4}$$

   and assumed the household maximized the present discounted flow of utility

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t^s\right) \tag{5}$$

   subject to the dynamic budget constraint

$$B_{t+1} = \left(1 + r_{t-1}\right) B_t + w_t L_t^s - C_t. \tag{6}$$

   (a) (4 points) Set up the Lagrangian (discrete-time Hamiltonian) that can be used to solve the household's maximization problem.

   (b) (4 points) Show that the resulting optimal savings condition is a Consumption Euler equation of the form

$$\ln C_{t+1} - \ln C_t = \ln\left(1 + r_t\right) + \ln \beta \approx r_t + \ln \beta \tag{7}$$

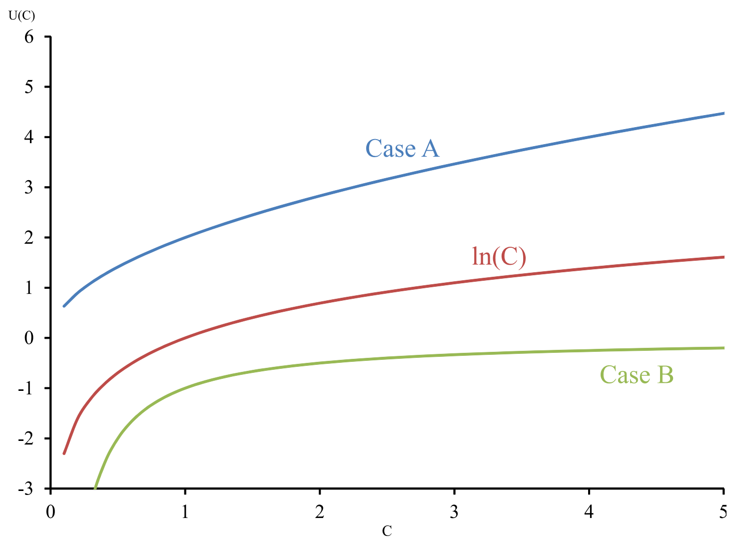
   (c) (4 points) What is the intertemporal elasticity of substitution for the households in this economy? How can it be gleaned from equation (7)?

   (d) (4 points) In the rest of this problem, we consider the case where utility does not depend on the labor supply, and denote utility by $U\left(C_t\right)$. What would a household with such preferences do in terms of its labor supply? What is this type of labor supply called?

(e) (4 points) Under such preferences, from part (d), the Consumption Euler equation boils down to

$$\frac{\partial U\left(C_t\right)}{\partial U\left(C_{t+1}\right)} = \beta\left(1 + r_t\right) \tag{8}$$

The figure below shows three utility functions. One is the logarithmic utility function and the other two are labeled A and B. Which utility function corresponds to a higher intertemporal elasticity of substitution than one? Explain why.



3. **Real business cycle facts (20 points)**

In class we discussed summary statistics of business cycle facts for the U.S. Table 1 reproduces the business cycle statistics for the United States. Based on the statistics in that table, answer the following questions. For each of your answers specfically point out which numbers from the table you base your answer on.

(a) (4 points) Are output deviations from trend in the U.S. persistent?

(b) (4 points) Do output, consumption, investment, and hours move together over the business cycle?

(c) (4 points) Are real wages procyclical?

(d) (4 points) In Germany the standard deviation of the cyclical fluctuations in consumption is 49% of that of output. Does this indicate that Germans have a higher or lower intertemporal elasticity of substitution than Americans?

(e) (4 points) Point out a fact in this table that the RBC model we studied in class has a hard time replicating.

Table 1: U.S. business cycle statistics

	Y	C	I	H hours	L employment	w
	Relative standard deviation (Y=100)					
	100	81	477	119	66	56
k	Correlation with Y (k quarters from now)					
4	0.11	0.28	0.20	-0.01	-0.07	0.09
3	0.34	0.47	0.39	0.20	0.12	0.14
2	0.61	0.66	0.58	0.45	0.35	0.16
1	0.85	0.81	0.76	0.70	0.60	0.19
0	1.00	0.84	0.88	0.87	0.80	0.17
-1	0.85	0.69	0.72	0.88	0.86	0.11
-2	0.61	0.47	0.48	0.76	0.79	0.05
-3	0.35	0.24	0.21	0.58	0.64	0.00
-4	0.12	0.02	-0.03	0.35	0.45	-0.06