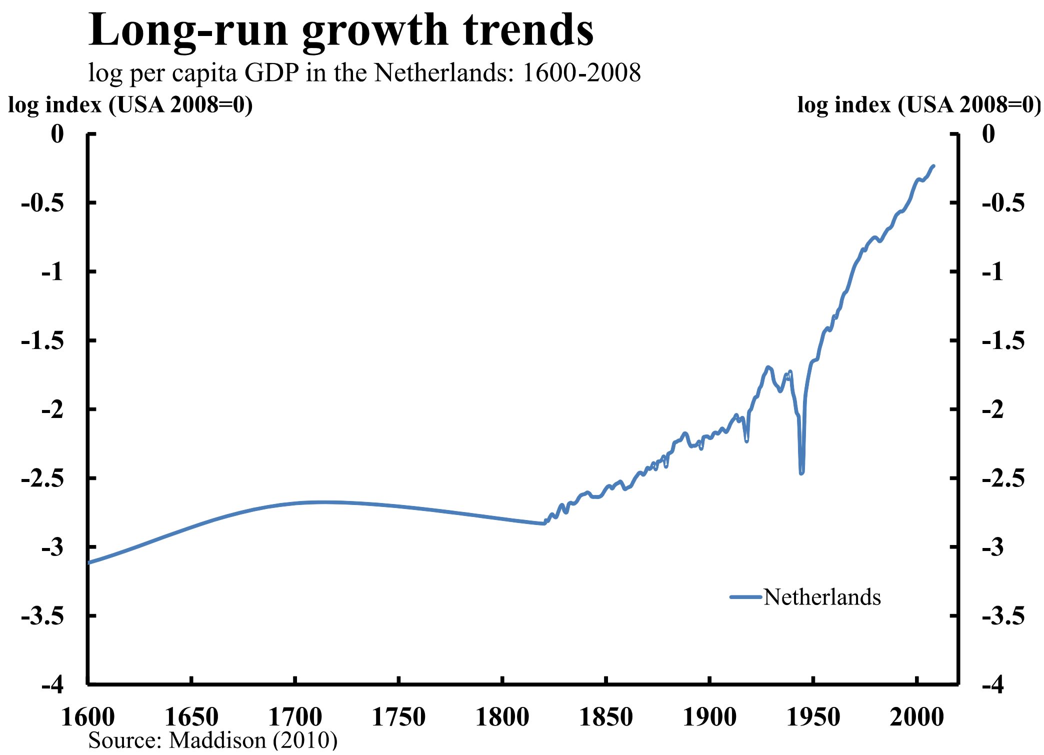


1. **The Dutch Growth Experience and the Solow and Ramsey-Cass-Koopmans models (20 points)**

   (a) 5p. The figure above shows the log of Dutch real GDP per capita from 1600-2008. Approximately, how many percent was the Dutch GDP per capita level lower than that in the United States in 2008. Explain how you can read this from the chart.

   Answer: The log-level of Dutch GDP per capita in 2008 is 0.2 lower than the log of that in the U.S. Since, for small values, log differences are approximately equal to percentage differences, this suggests that Dutch GDP per capita in 2008 was about 20 percent lower than that in the U.S.

   (b) 5p. Is the historical Dutch growth experience consistent with the transitional dynamics of the Solow growth model at a constant savings rate along a smooth technological trend? Why, or why not?

   Answer: It is not. The Solow growth model predicts transitional dynamics with high growth when GDP per capita is below its balanced growth path level and growth slowing down when it gets closer to its long-run trend. The historical experience in the Netherlands even sees a decline in GDP per capita after the Golden Age of the 18th century.

   (c) 5p.The transitional dynamics of the Ramsey-Cass-Koopmans model are different than those of the Solow model. Is the historical growth experience of the Netherlands consistent with that model? Why, or why not?

   Answer: It is not. Though, in the Ramsey-Cass-Koopmans model the savings rate is endogenous, its qualitative predictions about the growth path of economies are very similar to that of the Solow model. Just like the Solow model it predicts transitional dynamics with high growth when GDP per capita is below its balanced growth path level and growth slowing down when it gets closer to its long-run trend. Again, the Dutch growth experience is not consistent with this.

   (d) 5p. In the Solow model the savings rate is constant while in the Ramsey-Cass-Koopmans model it is determined by the equilibrium optimal savings condition

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha\left(1/k_t\right)^{1-\alpha} - \delta - \rho - \theta g}{\theta}.$$

Does this equation imply that savings rates are higher in countries with a high k_t or with a low k_t?

Answer: The above consumption Euler equation states that the growth rate of consumption is increasing in the real interest rate, which is the marginal product of capital corrected for depreciation, here $\alpha \left(1/k_t\right)^{1-\alpha} - \delta$. That is, when the real interest rate is high households consume less and save more, they put these savings towards future consumption and consumption will grow over time. As can be seen from the equation, the marginal product of capital is high when countries have a low k_t. Consequently, the savings rate is higher in countries with a low k_t.

2. **The intertemporal elasticity of substitution (20 points)**

In class, we solved the real business cycle model with log preferences, such that the utility function was

$$U\left(C_t, L_t^s\right) = \ln C_t + b \ln\left(1 - L_t\right) \tag{1}$$

and assumed the household maximized the present discounted flow of utility

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t^s\right) \tag{2}$$

subject to the dynamic budget constraint

$$B_{t+1} = \left(1 + r_{t-1}\right) B_t + w_t L_t^s - C_t. \tag{3}$$

(a) 3p. Set up the Lagrangian (discrete-time Hamiltonian) that can be used to solve the household's maximization problem.

Answer: The Lagrangian associated with the household's utility maximization problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^s \left[\ln C_t + b \ln\left(1 - L_t\right) + \lambda_t \left\{\left(1 + r_{t-1}\right) B_t + w_t L_t^s - C_t - B_{t+1}\right\}\right].$$

Here λ_t is the costate variable associated with the shadow value of addional asset holdings B_{t+1}. This can be used to derive the first-order conditions that give the optimal savings decision in part (b).

(b) 3p. Show that the resulting optimal savings condition is a Consumption Euler equation of the form

$$\ln C_{t+1} - \ln C_t = \ln\left(1 + r_t\right) + \ln\beta \approx r_t + \ln\beta \tag{4}$$

Answer: The optimality conditions that determine the optimal savings decision are

$$0 = \frac{\partial \mathcal{L}}{\partial B_{t+1}} = -\lambda_t + \beta \lambda_{t+1}\left(1 + r_t\right).$$

and

$$0 = \frac{\partial \mathcal{L}}{\partial C_t} = \frac{1}{C_t} - \lambda_t.$$

Combining these two optimality conditions, we obtain

$$\frac{C_{t+1}}{C_t} = \beta\left(1 + r_t\right).$$

Taking logarithms on both sides of the equation yields that

$$\ln C_{t+1} - \ln C_t = \ln\left(1 + r_t\right) + \ln\beta.$$

(c) 4p. What is the intertemporal elasticity of substitution for the households in this economy? How can it be gleaned from equation (4)?

Answer: The intertemporal elasticity of substitution measures the sensitivity of consumtion with respect to movements in the real interest rate. Note that $\ln(1+r_t) \approx r_t$, such that the above optimal savings decision reads

$$\ln C_{t+1} - \ln C_t \approx r_t + \ln \beta.$$

and a one percentage point increase in the real interest rate leads to a one percent increase in consumption in $t+1$ relative to period t. Hence, the intertemporal elasticity of substitution in this case equals one.

(d) 5p. In the rest of this problem, we consider the case where utility does not depend on the labor supply, and denote utility by $U(C_t)$. What would a household with such preferences do in terms of its labor supply? What is this type of labor supply called?

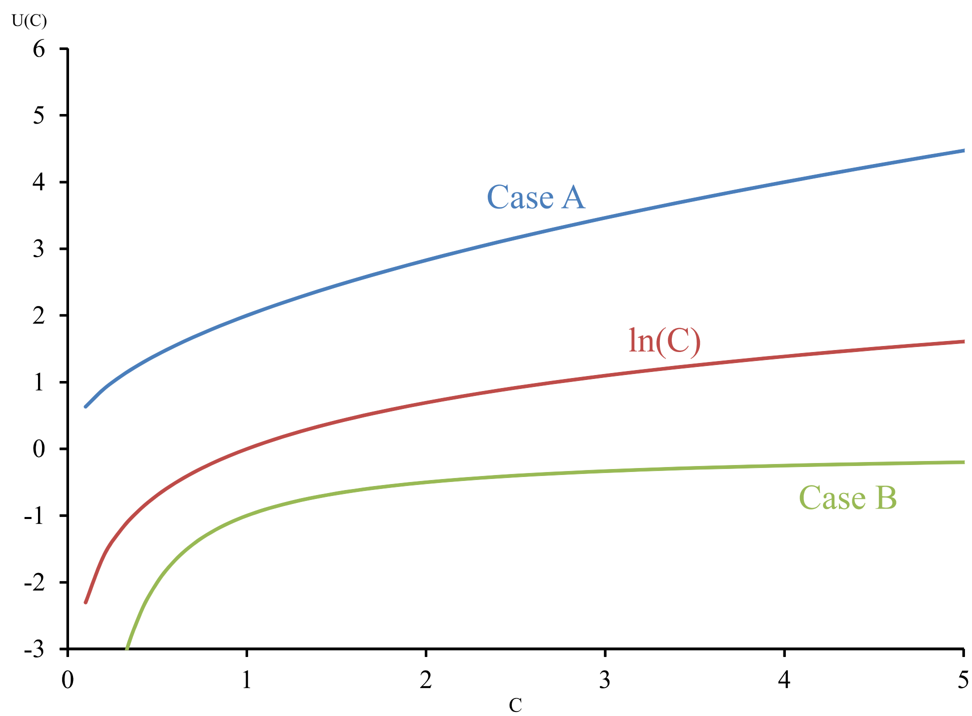
Answer: Since, in this case, the household does not get any utility loss from working, it will work as much as it can at any given positive wage. This is known as a perfectly inelastic labor supply. That is, the amount of labor supplied by the household does not respond to the level of the real wage at all.

(e) 5p. Under such preferences, from part (d), the consumption Euler equation boils down to

$$\frac{\partial U(C_t)}{\partial U(C_{t+1})} = \beta(1+r_t) \qquad (5)$$

The figure below shows three utility functions. One is the logarithmic utility function and the other two are labeled A and B. Which utility function corresponds to a higher intertemporal elasticity of substitution than one? Explain why.

Answer: The utility function that results in a larger intertemporal elasticity of substitution is the one in which C_t and C_{t+1} have to move a lot in order to satisfy the above equation when r_t changes. This is the case when $\partial U(C_t)/\partial U(C_{t+1})$ moves little in response to changes in C_t and C_{t+1}. That means that the second order derivative of the utility function is small and the utility function has less curvature. Hence, Case A corresponds to a higher intertemporal elasticity of substitution than the logarithmic utility function, i.e. higher than one. Case B, is the one with an intertemporal elasticity of substitution smaller than one.



3. **Finding the Neo-Classical back in the New Keynesian model (20 points)**

In class we derived the equations that define the equilibrium path of output, Y_t, hours, L_t, the real wage, W_t/P_t, inflation, π_t, the nominal interest rate, i_t, and the real interest rate, r_t, in the simple New Keynesian model of monetary policy. We discussed this model as a form of the New-Neo-Classical Synthesis. In this problem, you are asked to find back the Real Business Cycle part of the New-Keynesian model.

We considered these equations here with the addition of three shocks. The IS curve, derived from the household's savings decision reads

$$\ln Y_{t+1} = \frac{\ln \beta}{\theta} + \frac{1}{\theta} r_t + \ln Y_t - \ln D_t.$$

Here, D_t, is a demand shock. When it is positive the household will be more eager to substitute consumption towards the present and thus demand will go up. The upward-sloping AS-curve is generated by the New-Keynesian Phillips Curve

$$\pi_t = \left[\frac{\eta}{(\eta-1)}\frac{W_t}{P_t} - 1\right]\frac{\eta-1}{\xi}Y_t + \frac{1}{1+r_t}\pi_{t+1}.$$

The Monetary Policy Rule reads

$$i_t = \bar{i} + \gamma_\pi \pi_t + \gamma_y \left(\ln Y_t - \ln \overline{Y}\right) + \mu_t.$$

Here μ_t is the monetary policy shock. It reflects an unexpected deviation of the central bank from the policy rule to which it has credibly committed. The labor supply decision yields

$$\frac{W_t}{P_t} = B Y_t^\theta L_t^{\gamma-1},$$

while the Fisher identity links the real interest rate to the nominal interest rate and expected inflation.

$$i_t = r_t + \pi_{t+1}.$$

Finally, the production function reflects the technology with which output is produced. We assume a linear technology of the form

$$\ln Y_t = \ln L_t + \ln Z_t,$$

where $\ln Z_t$ reflects the shock to productivity. Here, the natural rate of interest and the natural rate of output are respectively given by

$$\bar{i} = \left(\frac{1}{\beta} - 1\right) \approx -\ln \beta \text{ and } \overline{Y} = \left(\frac{\eta-1}{\eta B}\right)^{\frac{1}{\theta+\gamma-1}}.$$

Each of the shocks follows an AR(1) process as defined in class. In this problem, we will analyze the properties of this equilibrium.

(a) 3p. What is the parameter that determines the degree of nominal rigidities (price stickiness) in this model?

Answer: The parameter that determines the level of price stickiness is ξ.

(b) 3p. What is the value of this parameter when prices are fully flexible?

Answer: When prices are flexible, then $\xi = 0$.

(c) 5p. Show that if prices are flexible, the equilibrium level of output equals

$$Y_t = \overline{Y} Z_t^{\frac{\gamma-1}{\theta+\gamma-1}}$$

Answer: When prices are flexible and $\xi = 0$, then firms set their prices according to a constant markup rule

$$\frac{W_t}{P_t} = \frac{\eta - 1}{\eta}$$

Substituting this and the production function into the labor supply condition, we obtain

$$\frac{\eta - 1}{\eta} = BY_t^\theta \left(\frac{Y_t}{Z_t}\right)^{\gamma - 1}.$$

Solving this for the level of output, Y_t, we obtain that

$$Y_t = \left(\frac{\eta - 1}{\eta B}\right)^{\frac{1}{\theta + \gamma - 1}} Z_t^{\frac{\gamma - 1}{\theta + \gamma - 1}}$$

(d) 3p. What is the value of the real interest rate in under flexible prices?

Answer: The value of the real interest rate in this case can be derived from the consumption Euler equation

$$r_t = \theta (\ln Y_{t+1} - \ln Y_t) - \ln \beta + \theta \ln D_t = \frac{\theta (\gamma - 1)}{\theta + \gamma - 1} (\ln Z_{t+1} - \ln Z_t) - \ln \beta + \theta \ln D_t$$

and is a function of the shocks but not of the monetary policy rule parameters.

(e) 3p. What is the value of the real wage under flexible prices?

Answer: From the derivation of the answer for part (c), you can see that under flexible prices the real wage equals

$$\frac{W_t}{P_t} = \frac{\eta - 1}{\eta}.$$

(f) 3p. When prices are flexible, does the allocation of goods and labor in this economy depend on the inflation rate? Discuss why this is similar to the Real Business Cycle model that we studied in class.

Answer: Under flexible prices the allocation of goods and labor in this economy is only a function of the shocks, Z_t and D_t, and preference and technology parameters. It is not influenced by the monetary policy decisions of the central bank. Hence, under flexible prices the Classical Dichotomy holds in this model, similarly to how it holds in the Real Business Cycle model.