1. **Equilibrium in the New-Keynesian model of monetary policy**

In class we derived the equations that define the equilibrium path of output, Y_t, hours, L_t, the real wage, W_t/P_t, inflation, π_t, the nominal interest rate, i_t, and the real interest rate, r_t. We consider these equations here with the addition of three shocks. The IS curve, derived from the household's savings decision reads

$$\ln Y_{t+1} = \frac{\ln \beta}{\theta} + \frac{1}{\theta} r_t + \ln Y_t - \ln D_t.$$

Here, D_t, is a demand shock. When it is positive the household will be more eager to substitute consumption towards the present and thus demand will go up. The upward-sloping AS-curve is generated by the New-Keynesian Phillips Curve

$$\pi_t = \left[\frac{\eta}{(\eta-1)}\frac{W_t}{P_t} - 1\right]\frac{\eta-1}{\xi}Y_t + \frac{1}{1+r_t}\pi_{t+1}.$$

The Monetary Policy Rule reads

$$i_t = \bar{i} + \gamma_\pi \pi_t + \gamma_y \left(\ln Y_t - \ln \overline{Y}\right) + \mu_t.$$

Here μ_t is the monetary policy shock. It reflects an unexpected deviation of the central bank from the policy rule to which it has credibly committed. The labor supply decision yields

$$\frac{W_t}{P_t} = B Y_t^\theta L_t^{\gamma-1},$$

while the Fisher identity links the real interest rate to the nominal interest rate and expected inflation.

$$i_t = r_t + \pi_{t+1}.$$

Finally, the production function reflects the technology with which output is produced. We assume a linear technology of the form

$$\ln Y_t = \ln L_t + \ln Z_t,$$

where $\ln Z_t$ reflects the shock to productivity. Here, the natural rate of interest and the natural rate of output are respectively given by

$$\bar{i} = \left(\frac{1}{\beta} - 1\right) \approx -\ln\beta \text{ and } \overline{Y} = \left(\frac{\eta-1}{\eta B}\right)^{\frac{1}{\theta+\gamma-1}}.$$

Each of the shocks follows an AR(1) process as defined in class. In this problem, we will analyze the properties of this equilibrium. Both analytically as well as using the computer to approximate the dynamic equilibrium path of the variables.

(a) What is the intertemporal elasticity of substitution in this model? What is its interpretation in terms of consumption smoothing? Do households with a high intertemporal elasticity of substitution smooth their consumption less or more than those with a low one?

Answer:

The intertemporal elasticity of substitution measures the responsiveness of the households' savings decision with respect to changes in the real interest rate. In terms of the parameters of the model, it is $1/\theta$. If $1/\theta$ is high than the households' savings decision is very sensitive to the real interest rate and the household smooths consumption less than when $1/\theta$ This can be seen from the IS-curve. If $1/\theta = 0$ and there are no demand shocks, such that $\ln D_t = 0$, then $\ln\beta/\theta = 0$ and consumption/output, Y_t, would be constant over time.

(b) What does the parameter B capture in this model? Explain why the steady-state level of output is declining in B.

Answer:

The parameter B reflects the level of the households' disutility of working. The higher B, the more utility is lost at the the same level of labor supply. The steady-state level of output is declining in B because in an economy in which households dislike working more, they will work less in equilibrium which, everything else equal, results in less output being produced.

(c) Show that if prices are flexible, the equilibrium level of output equals

$$Y_t = \overline{Y} Z_t^{\frac{\gamma-1}{\theta+\gamma-1}}$$

Answer:
Under flexible price setting the New-Keynesian Phillips curve does not hold. Instead, the firm will set its wage equal to a gross markup times its marginal cost. This means that, if $\xi = 0$, then the firms price-setting choice simplifies to

$$P_t = \frac{\eta}{(\eta-1)} W_t \text{ or } \frac{W_t}{P_t} = \frac{\eta-1}{\eta}.$$

Given that the production function is such that $L_t = Y_t/Z_t$. We can substitute these two equations in the household's labor supply condition to obtain

$$\frac{\eta-1}{\eta} = B Y_t^{\theta+\gamma-1} Z_t^{1-\gamma}$$

Solving this yields that

$$Y_t = \left(\frac{\eta-1}{\eta B}\right)^{\frac{1}{\theta+\gamma-1}} Z_t^{\frac{\gamma-1}{\theta+\gamma-1}},$$

which the expression we are asked to derive.

(d) What is the classical dichotomy between the real and nominal sides of the economy?

Answer:
The classical dichotomy between the real and nominal sides of the economy is the property of some macroeconomic models that the central bank's choice of policy rule and liquidity provision do not influence equilibrium quantities in the goods and labor market but only inflation. That is, the only thing affected by the supply of liquidity is the level of nominal variables but not that of quantities and relative prices.

(e) What does the result in part (c) imply about the classical dichotomy? Does it hold or not in this model? Why or why not?

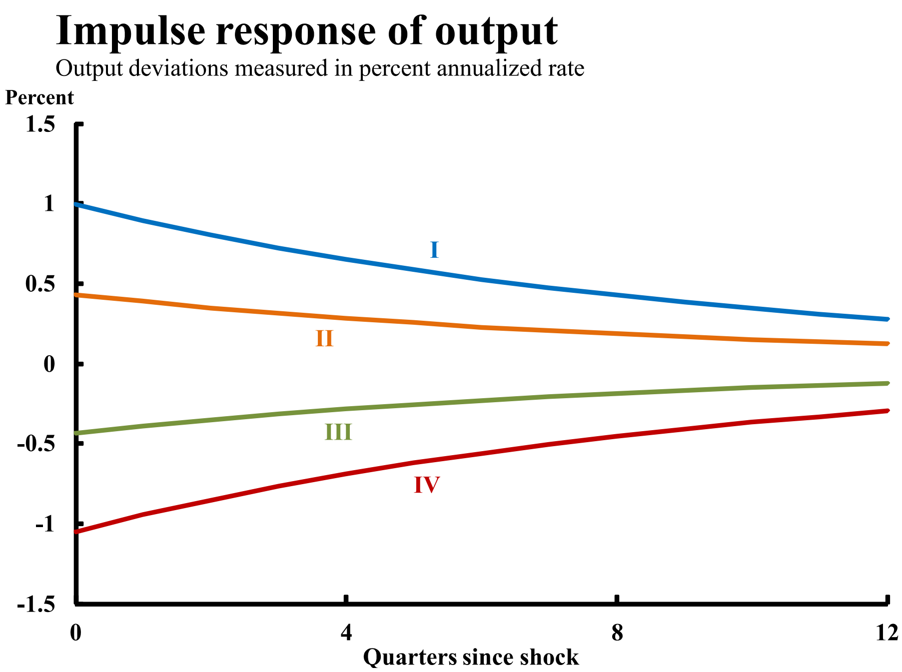
Answer:
Under flexible prices the classical dichotomy holds. As can be seen from part (c), if prices are flexible then the only thing that affects the level of output in equilibrium is the supply shock. Monetary policy has no effect on this level.

2. **Impulse response logikwiz**

In this problem we consider four particular cases for parameters and shocks for the model of the previous question. For all four cases β, B, η, γ_π, γ_y, ξ, μ_0, Z_0 and the persistence parameters of the shocks are the same. The differences in parameters and initial shocks across these cases are listed in Table 1. The figure plots four impulse responses of output. They are labeled I through IV. Each of these responses corresponds to one of the cases in Table 1. It is your job to figure out which responds to which. Explain the reason why the impulse responses are shaped the way they are.

Table 1: Parameters and shock differences across cases

	A	B	C	D
$1/\theta$	1	0.01	1	1
$1/(\gamma-1)$	0.5	0.5	0.5	10
$\ln D_0$	-1%	-1%	1%	1%



(a) Fill in Table 2 with zeros and pluses. To give an example, one element is already filled in. That is, case A can not be associated with impulse response function I because a negative demand shock would result in a decline in output and not an increase. Fill in all the rest of the elements in the same way. For each element either use an argument as the one in the example or derive it through deduction based on other elements derived in the table.

Table 2: Parameters and shock differences across cases

	A	B	C	D
I	-	-	-	+
II	-	-	+	-
III	+	-	-	-
IV	-	+	-	-

Answer:

Case A and B are either IRF III or IV: A negative demand shock would result in a decline in output relative to steady state. As a result, cases A and B, which are those with a negative demand shock, can not be IRFs I or II, which show an increase in output relative to steady state. Hence they have to be either III or IV.

Case A is IRF III: What distinguishes case A from case B is that case A is the one with the higher intertemporal elasticity of substitution. In this model, this means that when the central bank acts in response to a negative demand shock to stabilize output, then households with the higher elasticity of substitution react more strongly to the stabilization policy. As a result, in that economy the central bank is able to stabilize output more than in the economy with the low elasticity of substitution. In IRF III the output response to the negative demand shock is less than in IRF, output is stabilized more. Hence, case A is IRF III.

Case B is IRF IV: This follows by deduction from the two parts above.

Case C is IRF II: The difference between cases C and D is that in case D the Frisch elasticity of the labor supply is much higher than in case C. Hence, in case D the labor supply of households increases much more in response to an increase in the real wage than in case C. Hence, for a given level of the productivity shock, an increase in output can be achieved at much less of an increase in the marginal cost of production under case D than under case C. Thus, a positive demand shock in case D is much less inflationary and the central bank will act much less to offset. As a result, the output response to a positive demand shock under case D is higher than under case C. Thus case C corresponds to the IRF with the smaller output response; IRF II.

Case D is IRF I: Assuming the above logic is correct, this is the only option left.