Question 1: Monetary Policy Logikwiz

This question will always be graded. It can not be the one you skip on the exam.

We consider a version of the three-equation New-Keynesian model that we studied in class and add both a demand as well as a supply shock to the model. The IS-curve in the model is given by the Euler equation implied by the household sector's savings decision and reads.

$$Y_t = Y_{t+1} \frac{D_{t+1}}{D_t} \frac{1}{\beta} \frac{1+\pi_{t+1}}{1+i_{t+1}}. \tag{1}$$

Here D_t is a demand shock which is 1 in steady state. We consider percentage deviations of the demand shock from its steady-state value, such that

$$\widehat{D}_t = \frac{D_t - 1}{1} = D_t - 1 \neq 0 \tag{2}$$

We allow for persistence in this shock and will assume that percentage deviations of D_t from its steady state level of one, i.e. $\widehat{D}_t$, follows

$$\widehat{D}_t = \rho_D \widehat{D}_{t-1}, \text{ where } |\rho_D| < 1 \tag{3}$$

This is a demand shock, because it affects the discount factor of the household. A higher D_t essentially reflects a temporary decrease in the discount factor, β. Therefore, if D_t is positive, the households value current consumption more than future consumption than in steady state, they will thus save less and consume more.

Firms are monopolistic competitors that each face the production function $Y_t = Z_t L_t$. Here Z_t is the productivity level of the firms. They face a quadratic adjustment cost for their prices equal to ξ and an own price elasticity of demand equal to $\eta > 1$. In the symmetric equilibrium and solving out for the equilibrium level of the real wage we obtain that the firms decide on their prices using the following intertemporal optimality condition:

$$\pi_t = \left[\frac{\eta}{(\eta-1)} \frac{Z_t Y_t}{Z_t - Y_t} - 1 \right] \frac{\eta - 1}{\xi} Y_t + \frac{1}{1+r_t} \pi_{t+1}. \tag{4}$$

In steady state $Z_t = 1$. However, we consider the effect of a shock, and look at percentage deviations from steady state. These are defined as

$$\widehat{Z}_t = \frac{Z_t - 1}{1} = Z_t - 1 \neq 0 \tag{5}$$

We allow for persistence in this shock and assume that percentage deviations of Z_t from its steady state level of one, i.e. $\widehat{Z}_t$, follow

$$\widehat{Z}_t = \rho_z \widehat{Z}_{t-1}, \text{ where } |\rho_z| < 1. \tag{6}$$

This is a supply shock, because it affects the production structure of the economy and, a positive shock to TFP, will lead to a reduction in the unit production costs.

The central bank is assumed to credibly pursue the policy rule

$$i_{t+1} = \bar{r} + \gamma_\pi \pi_t + \gamma_y \left(\ln Y_t - \ln \overline{Y}_t \right). \tag{7}$$

Here $\bar{r}$ is the steady-state real interest rate and $\ln \overline{Y}_t$ is the log of the steady-state output level.

We consider five parameter combinations. They are listed in Table 1 below. We have a benchmark model and four cases of alternative parameter values.

Table 1: Benchmark and alternative parameter values

			Case			
Parameter	**Interpretation**	**benchmark**	**(I)**	**(II)**	**(III)**	**(IV)**
β	Discount factor	0.99	0.99	0.99	0.99	0.99
η	Own price elasticitiy of demand	11	11	11	11	11
ξ	Adjustment cost of prices	2	2	0.1	2	2
γ_π	Taylor rule inflation parameter	1.25	1.25	1.25	1.25	6
γ_y	Taylor rule output gap parameter	0.25	0.25	0.25	0.25	0.25
ρ_D	Persistence of demand shock	0.5	0.5	0.5	0.9	0.5
ρ_Z	Persistence of supply shock	0.5	0.5	0.5	0.5	0.5
$\widehat{D}_0$	Initial demand shock	1%	0%	1%	1%	1%
$\widehat{Z}_0$	Initial supply shock	0%	0.1%	0%	0%	0%

Figure 1 plots the impulse responses for the benchmark model as well as the four different parameter combinations. However, these parameter combinations are not labeled by their roman numerals. Instead, they are relabeled models (a) through (d). The aim of this question is for you to figure out which model corresponds to which case of parameter values.

Answer: This question is easiest answered by deduction. Parts (a) and (c) are relatively straightforward. This leaves the choice between case (II) or case (IV) for part (b).

(a) To which of the four cases (I) through (IV) of the parameter values do the impulse responses of model (a) correspond? Explain your answer.

Answer: *For Model (a) the deviations from the steady state are much more persistent than any of the other models. This must be the case of the highly persistent demand shock. That is case (III).*

(b) To which of the four cases (I) through (IV) of the parameter values do the impulse responses of model (b) correspond? Explain your answer.

Answer: *For case (II) the adjustment cost of prices is almost zero. This means that a demand shock is relatively more inflationary and does not generate any output response. This means that Model (d) is Case (II). Since, we have pinned down models (a) and (c) already, it must be that model (b) is Case*

(IV). When the inflation parameter in the Taylor rule is higher the central bank raises the nominal interest rate more in response to a demand shock. The result is less inflation (and lower inflation expectations). However, this raises the real interest rate and reduces aggregate demand through the IS curve. The result is a lower increase in output than would have occurred for a higher γ_π. *This is consistent with the path of the equilibrium variables plotted for model (b). Hence model (b) is Case (IV).*

(c) To which of the four cases (I) through (IV) of the parameter values do the impulse responses of model (c) correspond? Explain your answer.

Answer: *In the case of Model (c) we are moving in the opposite direction of the output-inflation trade-off. That is, output increases while inflation goes down. This must be the case of a positive productivity shock, i.e. case (I)*

Scoring: 3 out of 3 (100 points), 2 out of 3 (60 points), 1 out of 3 (30 points).

The Government Budget Constraint and Ricardian Equivalence

We considered two dynamic budget constraints. The first was the household sector's budget constraint

$$\int_{t=0}^{\infty} e^{-R(t)} C(t)\, dt = K(0) + D(0) + \int_{t=0}^{\infty} e^{-R(t)} [W(t) - T(t)]\, dt.$$

The second was the government's budget constraint

$$\int_{t=0}^{\infty} e^{-R(t)} G(t)\, dt = -D(0) + \int_{t=0}^{\infty} e^{-R(t)} T(t)\, dt.$$

Here

- $K(0)$ is the initial capital stock in the economy
- $D(t)$ real government debt at time t
- $C(t)$ real consumption purchases at time t
- $G(t)$ real government purchases at time t
- $T(t)$ real net tax revenue at time t
- $r(t)$ real interest rate at time t
- $W(t)$ is real income of the household sector at time t
- $R(t) = \int_{\tau=0}^{t} r(\tau)\, d\tau$ is the compounded discount factor at time t

(a) What does $D(0)$ represent in both the household sector's budget constraint as well as in the government's budget constraint?

Answer: *$D(0)$ is the initial level of government debt that the government sector owes to the household sector.*

(b) The government's budget constraint is derived under a No-Ponzi condition. What is this condition? Write down its algebraic expression and explain the intuition behind it.

Answer: *The No-Ponzi condition reads*

$$\lim_{t\to\infty} e^{-\int_0^t r(s)ds} D(t) = 0. \tag{8}$$

It reflects that the government can not continue to rollover debt to pay for its outstanding debt. If it did, the present discounted value of its repayments to its current debtors would be effectively zero. By continuously kicking the debt can down the road, the government essentially never pays back its outstanding debt. If those who supply the debt realize that they will basically never get paid back, they will stop providing funding to the government.

(c) Suppose the government is indebted at time 0 such that $D(0) > 0$ and it is planning on never running a primary surplus, that is $G(t) \geq T(t)$ for all $t > 0$. Can this government ever satisfy the No-Ponzi condition? Why or why not?

Answer: *The answer to this question is best understood by realizing that the government budget constraint is derived from integrating the following debt accumulation equation*

$$\underbrace{\overset{\bullet}{D}(t)}_{\text{change in government debt}} = \underbrace{\underbrace{[G(t) - T(t)]}_{\text{primary deficit}} + r(t) D(t)}_{\text{budget deficit}}. \tag{9}$$

subject to the No-Ponzi condition. A government with $D(0) > 0$ and $G(t) > T(t)$ for all $t > 0$ will violate the No-Ponzi condition. To see this, we can use the solution that

$$D(t) = \int_0^t e^{\int_s^t r(s)ds} (G(s) - T(s))\, ds + e^{\int_0^t r(s)ds} D(0),$$

such that

$$e^{-\int_0^t r(s)ds} D(t) = \int_0^t e^{-\int_0^s r(s)ds} (G(s) - T(s))\, ds + D(0).$$

Hence, if $G(t) - T(t) > 0$ for all $t > 0$ then it must be the case that

$$\lim_{t\to\infty} e^{-\int_0^t r(s)ds} D(t) > D(0) > 0,$$

which is a violation of the No-Ponzi condition. This is the case because the government never runs a primary surplus to pay off its interest and debt. In this case the government continuously finances it current debt by issuing new debt... A classic Ponzi scheme.

(d) Combine the above two budget constraints to obtain the aggregate budget constraint for the whole economy.

Answer: *The aggregate budget constraint can be derived by realizing that*

$$\begin{aligned}
\int_{t=0}^{\infty} e^{-R(t)} C(t)\, dt &= K(0) + D(0) + \int_{t=0}^{\infty} e^{-R(t)} [W(t) - T(t)]\, dt \\
&= K(0) + \int_{t=0}^{\infty} e^{-R(t)} W(t)\, dt - \left[\int_{t=0}^{\infty} e^{-R(t)} T(t)\, dt - D(0)\right] \\
&= K(0) + \int_{t=0}^{\infty} e^{-R(t)} W(t)\, dt - \int_{t=0}^{\infty} e^{-R(t)} G(t)\, dt,
\end{aligned}$$

such that

$$\int_{t=0}^{\infty} e^{-R(t)} C(t)\, dt + \int_{t=0}^{\infty} e^{-R(t)} G(t)\, dt = K(0) + \int_{t=0}^{\infty} e^{-R(t)} W(t)\, dt.$$

(e) Use this aggregate budget constraint to explain why the choice whether to finance current government expenditures through the issuance of debt or the generation of tax revenue does not matter for the overall level of economic activity in this economy.

Answer: *This budget constraint shows that the households' lifetime budget constraint does not depend on the way the government finances it purchases. It only depends on the present discounted value of government purchases, i.e. on* $\int_{t=0}^{\infty} e^{-R(t)} G(t)\, dt$*. Because the households' budget constraint does not depend on the way the government finances its spending neither do the households' consumption decisions.*

(f) Discuss three different assumptions that deviate from the framework above under which the government's financing decision of its spending would affect economic activity. Be specific about which assumptions that we made to derive the result above would be violated and whether debt financing of government expenditures would increase or decrease overall economic activity if those assumptions are violated.

Answer: *Here are three of many possible violations of the assumptions: (i) the derivation assumes that the households and the government pay the same interest rate on their debt. In reality governments tend to pay lower rates than households and businesses. In this case deficit spending by the government has the potential to increase overall economic activity. (ii) The derivation assumes that households and governments are infinitely lived and forward looking. In an overlapping generations setup where parents discount their kids future higher than their own, deficit spending could increase consumption since the parents do not care that much about their children having to pay off the currently incurred government debt. (iii) We have assumed that government spending has no effect on the marginal product of labor and thus wages. Spending on infrastructure, for example, seems to increase economywide productivity and would actually affect the righthand side of the overall budget constraint.*

Scoring: (a) 10, (b) 20, (c) 20, (d) 10, (e) 20, (f) 20.

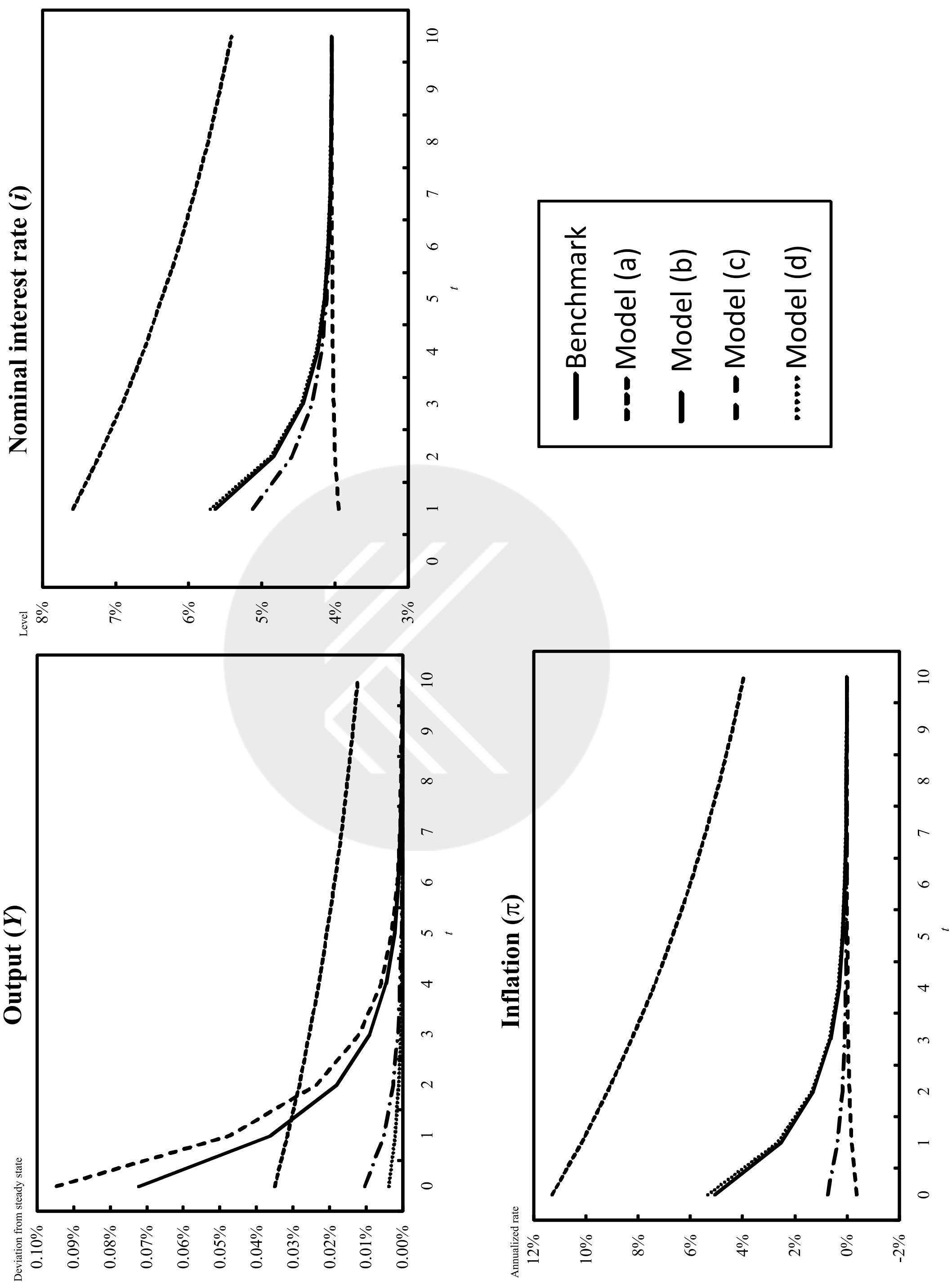

Figure 1: Impulse responses for Benchmark and four cases.